Hot-wire pyrometry

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A hot-wire pyrometer for measuring gas temperatures with a high sensitivity has been developed. The method can easily be extended to measurements of surface temperature. The principal elements of a hot-wire pyrometer are a fine wire inserted into the gas and a photomultiplier for measuring the radiation emitted from it. The output voltage V of the photomultiplier is shown both theoretically and experimentally to be the following function of wire temperature T: $V=\sigma T^{7/4}\exp(-\gamma T^{-1/2})$, where σ and γ are the calibration constants unique to a particular system.

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The hot-wire pyrometer was developed to measure the temperature-distance curve of a laminar flame. The signal from a hot-wire pyrometer depends on temperature to a power of about 14, so that it is particularly useful for measuring small temperature changes.

The basic elements of the instrument are a wire and a photomultiplier. A typical experimental setup is shown in Fig. 1. Here, a wire of temperature T is imaged on the entrance slit of a photomultiplier. The spectral flux of the photons from the wire per unit area is

$$J_\nu=\frac{2\pi\epsilon_\nu\nu^2}{c^2}\frac{1}{\exp(h\nu/kT)-1}, \tag{1}$$

where h is Planck's constant, ν is the frequency, c is the speed of light, k is Boltzmann's constant, and ϵ_ν is the spectral emmissivity of the wire.

The photon flow per unit wire area incident on the photomultiplier cathode is proportional to J_ν times the optics transmittance T_ν integrated over all frequencies.

The constant of proportionality is called the shape factor of the wire surface area A_W imaged on the photo multiplier slit to the area of the lens A_L. The shape factor $F_{W\text{-}L}$ is a purely geometric property of the system. The rate at which photoelectrons are emitted by the photosensitive material is

$$\dot{N}_e=A_W F_{W\text{-}L}\int_0^\infty J_\nu T_\nu Q_\nu\, d\nu. \tag{2}$$

A characteristic of many photomultipliers is that for small enough frequency, the quantum efficiency is an exponential function of frequency of the form

$$Q_\nu=\alpha\exp(-\beta/\nu),\quad \nu<\nu_0. \tag{3}$$

To make use of Eq. (3), a filter is employed so that the photomultiplier sees only frequencies $\nu<\nu_0-\delta\nu_*$, where $\delta\nu_*$ is a small frequency shift to be defined later. The transmittance of the optics is modeled as

$$\begin{aligned} T_\nu &= T_0, \qquad \nu\leqslant\nu_f,\\ &=0, \qquad \nu>\nu_f. \end{aligned} \tag{4}$$

Substitution of Eqs. (1), (3), and (4) into Eq. (2) yields an integral that cannot be solved in closed form even when ϵ_ν is independent of frequency. It can, however, be solved asymptotically the the method of Laplace.[1]

To proceed with a solution, it is convenient to define a function K_ν, so that Eq. (2) can be written in a form standard for the application of Laplace's method. That function is

$$K_\nu=[1-\exp(-h\nu/kT)]^{-1}, \tag{5}$$

and the photoelectric emission rate is

$$\dot{N}_e=A_W F_{W\text{-}L}(2\pi/c^2)\alpha T_0 \times\int_0^{\nu_f}\epsilon_\nu K_\nu\nu^2\exp[(-h\nu/kT)-(\beta/\nu)]\,d\nu. \tag{6}$$

The asymptotic solution to Eq. (6) is

$$\dot{N}_e\sim A_W F_{W\text{-}L}(2\pi/c^2)\,\alpha T_0\epsilon_{\nu *}\nu_*^2\exp(-2\beta/\nu_*)\,\delta\nu_* \quad (\delta\to 0), \tag{7}$$

$$\nu_*=(\beta kT/h)^{1/2}, \tag{8}$$

$$\delta=(\pi^2kT/\beta h)^{1/4}. \tag{9}$$

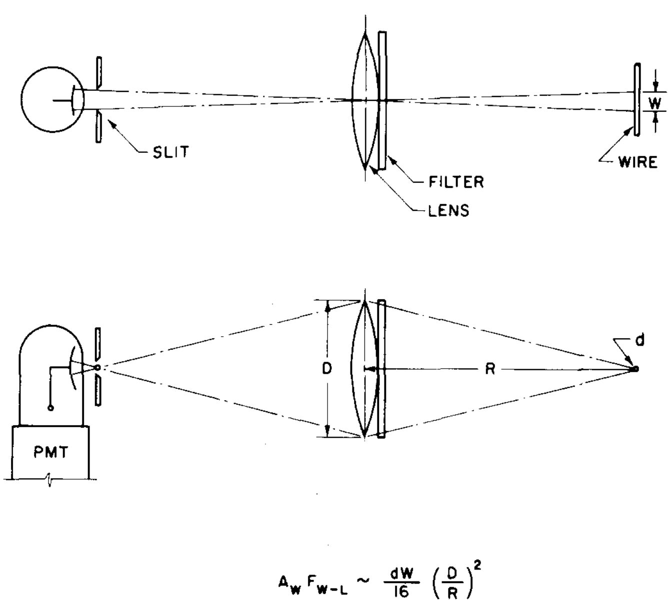

FIG. 1. An example of a hot-wire pyrometer.

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TABLE I. Experimental parameters and theoretical calibration constants. The data is given in c.g.s. units. $\epsilon = 0.22$. $T_0 = 0.8$; $\nu_f = 5.2\times10^{14}$; $\nu_0 = 5.6\times10^{14}$; $R = 1.3\times10^5$.

System	PMT	α	β	G	$A_W F_{W\text{-}L}$	σ	γ
1	1P28	6.2×10^3	6.7×10^{15}	9×10^4	3.0×10^{-6}	5×10^6	1130
2	1P21	9.6×10^4	8.1×10^{15}	8×10^4	6.0×10^{-7}	9×10^7	1250

The system acts like a narrow-band-pass filter with a peak transmittance at a frequency equal to ν_* and a band pass equal to the product $\delta\nu_*$. The photoelectric emission rate is a strong function of wire temperature for two reasons: (1) The spectral intensity at ν_* is a strong function of temperature and (2) the frequency of peak response ν_* is itself temperature dependent, so that the quantum efficiency is an increasing function of temperature.

The voltage across the load resistor of the photomultiplier is

$$V = \dot{N}_e eGR, \tag{10}$$

where e is the change per electron, G is the photomultiplier gain, and R is the load resistance. Therefore, for constant $\epsilon_{\nu_*} = \epsilon$, the following relationship exists between the output signal and the wire temperature:

$$V = \sigma T^{7/4} \exp(-\gamma T^{-1/2}), \tag{11}$$

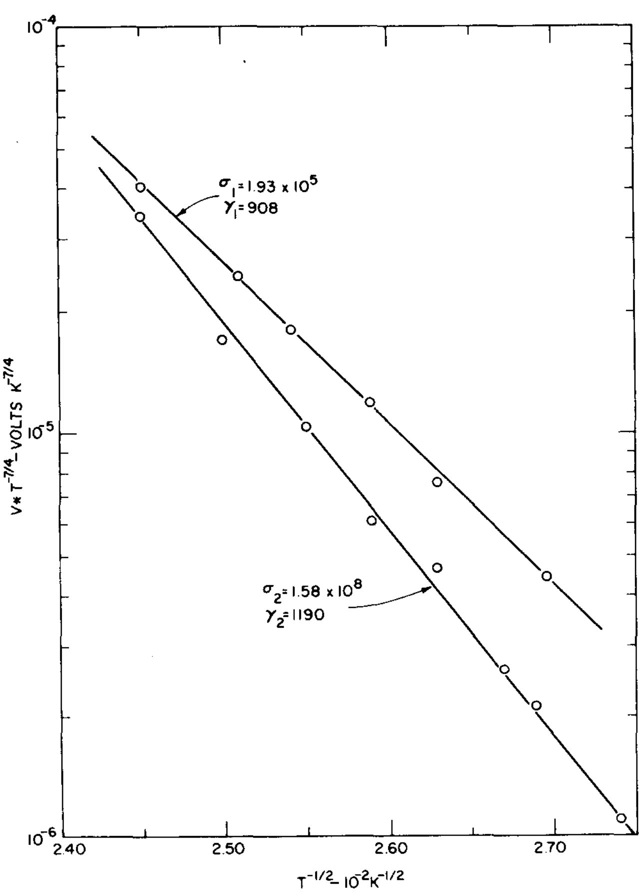

FIG. 2. Calibration curves for the two hot-wire pyrometers detailed in Table I.

$$\sigma = (\beta k/h)^{7/4} (\pi/\beta)^{1/2} (2\pi/c^2) \alpha T_0 \epsilon A_W F_{W\text{-}L} eGR, \tag{12}$$

$$\gamma = 2(h\beta/k)^{1/2}. \tag{13}$$

Equation (11) has been tested for two systems by using a silica-coated platinum: platinum-13% rhodium thermocouple for the hot wire. The wire was heated to different temperatures by a laminar flame. The small background signal from the flame radiation was subtracted from the measured intensities. The two systems had different photomultipliers and different shape factors. The experimental data is given in Fig. 2, and the theoretical calibration constants and system parameters are given in Table I. The photomultiplier constants α, β, and G were estimated from the manufacturer's typical specifications. In light of that uncertainty, quantitative agreement has been realized between theory and experiment.

Theoretically, the calibration curve is limited to temperatures such that the dimensionless band pass δ is small compared to unity. For flame measurements, the temperatures are limited to less than 1800 K by the silica coating[2] that is necessary to kill catalytic heating of the wire. It is also limited by either photomultiplier dark current or flame radiation to temperatures greater than about 900 K.

In application, one may also want to consider that the radiative heat flow from the wire must be balanced by convection of heat from the gas to the wire. The wire temperature is therefore slightly less than that of the gas. To compute the gas temperature in most flows, the following formula will usually suffice:

$$T_{\text{gas}} = T + (\epsilon\sigma_b T^4 d/2\lambda) \ln(4.492/P_0), \tag{14}$$

where σ_b is the Stefan-Boltzmann constant, λ is the thermal conductivity of the gas, and P_0 is the Peclet number based on wire diameter, gas speed, and on the thermodiffusivity of the gas. Equation (14) is valid for small Peclet numbers.[3]

[1] E. T. Copson, *Asymptotic Expansions* (Cambridge U.P., Cambridge, England, 1971), p. 36.
[2] W. E. Kaskan, *Sixth Symposium (International) on Combustion* (Reinhold, New York, 1957), p. 134.
[3] C. R. Illingworth, in *Laminar Boundary Layers*, edited by L. Rosenhead (Oxford U.P., London, 1963), p. 163.