Feynman
High Energy Phen

James Keck
Rockefeller Hall
Cornell U.
Ithaca, N.Y.

NATIONAL

Tel 6597

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THIS BOOK CONTAINS EYE-EASE PAPER
"Easy on the Eyes"

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Discussion of Dirac Equation Free particle

$$\left[\frac{\hbar}{i}\alpha\cdot\nabla + \beta\mu\right]\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} ; \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\gamma_1 \equiv \gamma_x \equiv \beta\alpha_x$$

$$\cdots \qquad \cdots$$

$$\gamma_4 \equiv \gamma \equiv \beta$$

The γ's transform like a 4 vector.

If a_μ is a 4-vector

$a_\mu \gamma_\mu \equiv \not{a}$ is an invariant

$$= a_4\gamma_4 - a_1\gamma_1 - a_2\gamma_2 - a_3\gamma_3$$

$$\not{a}\gamma_4 + \gamma_4\not{a} = 2a_4$$

$$\not{a}^2 = a_4^2 - a_1^2 - a_2^2 - a_3^2 = a_\mu a_\mu$$

$$a_\mu\gamma_\mu b_\nu\gamma_\nu + b_\nu\gamma_\nu a_\mu\gamma_\mu = a_\mu b_\nu(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu)$$
$$= 2a_\mu b_\mu$$

(The γ's anti commute $(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) = 2g_{\mu\nu}$

$$g_{\mu\nu} = \begin{pmatrix} I & & & 0 \\ & -I & & \\ & & -I & \\ 0 & & & -I \end{pmatrix})$$

1

$$\not{a}\not{b} + \not{b}\not{a} = 2a\cdot b$$
$$\not{a}\not{b} = 2a\cdot b - \not{b}\not{a}$$

Dirac Eq in terms of γ's

$$\left(\frac{\hbar}{i}\beta\alpha\cdot\nabla + \frac{\hbar}{i}\beta\frac{\partial}{\partial t}\right]\Psi = -\mu\Psi$$

$$-\frac{\hbar}{i}\gamma_\mu\frac{\partial}{\partial x_\mu}\Psi = \mu\Psi$$
$$\equiv$$
$$i\not{\nabla}\Psi = \mu\Psi$$

Solution of Free Particle Eq.

$$\Psi = u e^{-i p_\mu x_\mu}$$

$$\gamma_\mu p_\mu u = \mu u = \not{p} u$$
$$\not{p}^2 u = \mu \not{p} u = \mu^2 u$$
$$\not{p}^2 = \mu^2$$
$$E^2 - \underline{p}\cdot\underline{p} = \mu^2$$

Light: $e^{-i(\omega t - \underline{k}\cdot\underline{x})} = e^{-i k_\mu x_\mu}$

where $\omega^2 = \underline{k}\cdot\underline{k}$

$k^2 = k_\mu k_\mu = \omega^2 - \underline{k}\cdot\underline{k} = 0$

D.E. with Fields

$\frac{\hbar}{i}\nabla - \frac{e}{c}\underline{A}, \frac{\hbar}{i}\frac{\partial}{\partial t} + e\varphi.$

$(i\nabla\!\!\!/ - \frac{e}{c}A\!\!\!/)\psi = \mu\psi$

or

$(i\nabla\!\!\!/ - \mu)\psi = \frac{e}{c}A\!\!\!/\psi$

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Feynman Formulation:

$(i\nabla\!\!\!/ - \mu)\psi = A\!\!\!/\psi$

Consider

$(i\nabla\!\!\!/ - \mu)\psi(1) = f(1)$

If $K(1,2)$ is a solution of

$(i\nabla\!\!\!/ - \mu)K_+(12) = i\delta(1,2)$

$(i\nabla\!\!\!/ - \mu)\int K_+(12)f(2)d\tau_2 = \int i\delta(1,2)f(2)d\tau_2$

$= if(1)$

Hence

$$\psi(1) = -i\int K_+(12)\, f(2)\, d\tau_2 + \phi(1)$$

where

$$(i\nabla\!\!\!/ - \mu)\,\phi(1) = 0$$

$$\psi(1) = -i\int K_+(12)\, A\!\!\!/(2)\, \psi(2)\, d\tau_2 + \phi(1)$$

If $A\!\!\!/$ small

$$\psi(1) = \phi(1) - i\int K_+(12)\, A\!\!\!/(2)\, \phi(2)\, d\tau_2 - \iint K_+(12)\, A\!\!\!/(2)\, K_+(23)\, A\!\!\!/(3)\, \phi(3)\, d\tau_2\, d\tau_3 + i\iiint \dots$$

<u>Matrix elements:</u>

$$\langle \phi_f\, \psi \rangle = \int \phi_f^*(1)\, \phi_i(1)\, d\tau_1$$

$$- i\iint \phi_f^*(1)\, K_+(12)\, A\!\!\!/(2)\, \phi_i(2)\, d\tau_2\, d\tau_1$$

$$- \dots$$

$$= 0 - i\int \phi_f^*(2)\, A\!\!\!/(2)\, \phi_i(2)\, d\tau_2$$

$$- \iint \phi_f(2)\, A\!\!\!/(2)\, K_+(23)\, A\!\!\!/(3)\, \phi(3)\, d\tau_2\, d\tau_3$$

$$+ i \dots$$

Alternative Treatment

$$\psi = \frac{1}{i\nabla - \mu} A\psi + \phi$$

$$= \phi + \frac{1}{i\nabla - \mu} A\phi + \frac{1}{i\nabla - \mu} A \frac{1}{i\nabla - \mu} A\phi \ldots$$

Transformation to momentum space:

$$A(x_\mu) = \int A(q)\, e^{-iq_\mu x_\mu}\, d^4q$$

$$\nabla \to \gamma_\mu \frac{\partial}{\partial x_\mu} \quad ; \quad \phi = u\, e^{-ip_\mu x_\mu}$$

$$\frac{\partial}{\partial x_\mu} e^{-i\pi_\mu x_\mu} = (-i\pi_\mu)\, e^{-i\pi_\mu x_\mu}$$

$$\frac{1}{i\nabla - \mu} A\phi = \int \frac{1}{p + q - \mu} A(q)\, u\, e^{-i(p_\mu + q_\mu)x_\mu}\, dq$$

$$\frac{1}{i\nabla - \mu} A \frac{1}{i\nabla - \mu} A\phi = \iint \frac{1}{p + q_1 + q_2 - \mu} A(q_2) \frac{1}{p + q_1 - \mu} A(q_1)\, u\, d^4q_1\, d^4q_2\, e^{-i(p_\mu + q_{\mu_2} + q_{\mu_1})x_\mu}$$

$$\frac{1}{\pi - \mu} \equiv \frac{\pi + \mu}{\pi^2 - \mu^2}$$

$p_f = p_i + q$

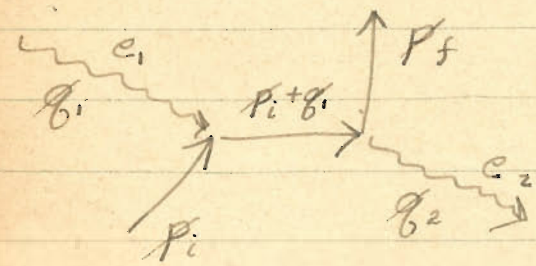
matrix elements:

$(\tilde{u}_f \not{A} u_i)$: $p_f = p_i + q$

$(\tilde{u}_f \not{A}(q_2) \frac{1}{\not{p} + \not{q}_1 - \mu} \not{A}(q_1) u_i)$: $p_f = p_i + q_1 + q_2$

This is the formulation used for computing.

Compton Effect

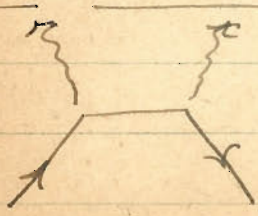


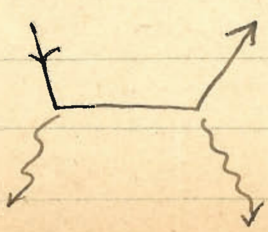
$p_f = p_i + q_1 - q_2$

$u_f (\not{e}_2 \frac{1}{\not{p}_i + \not{q}_1 - \mu} \not{e}_1) u_i$

$+ u_f (\not{e}_1 \frac{1}{\not{p}_i - \not{q}_2 - \mu} \not{e}_2) u_i$

Pair Production

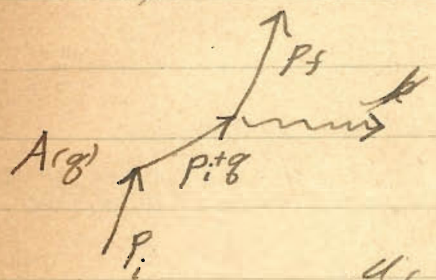




These processes are similar to Compton Effect except sign of momentum is reversed for positron.

Bremsstrahlung

$$\not{A} = \int \not{A}(q)\, e^{iq\cdot x}\, dq$$



$$p_f = p_i + q - k$$

$$\bar{u}_f \left(\not{e} \frac{1}{\not{p}+\not{q}-\mu} \not{A}(q) \right) u_i$$

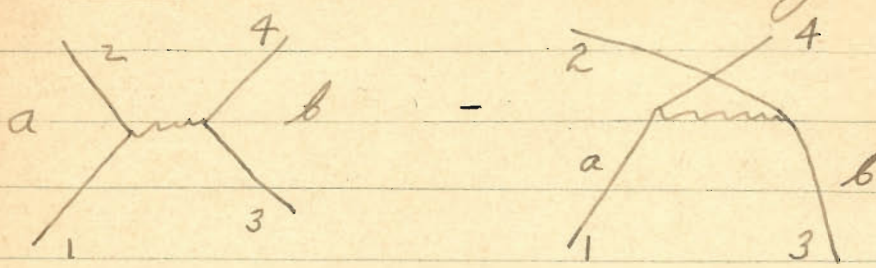
$$+\ \bar{u}_f \left(\not{A}(q) \frac{1}{\not{p}-\not{k}-\mu} \not{e} \right) u_i$$

For a stationary nucleus

$$V = \frac{Ze^2}{r}$$

$$\not{A}(q) = \gamma_4\, \delta(q_4)\, \frac{Ze^2 4\pi}{q\cdot q}$$

Electron - Electron Scattering



$$p_2 - p_1 = q = p_4 - p_3$$

$${}_4(\gamma^b_\mu)_3\,{}_2(\gamma^a_\mu)_1\,\frac{1}{q^2} - {}_1(\gamma^a_\mu)_4\,{}_3(\gamma^b_\mu)_2\,\frac{1}{q^2}$$

Alternative treatment: We may say the electron b constitutes a current which is acted upon by the potential, A, generated by electron a.

For scattering by $A_\mu = a_\mu e^{-iq\cdot x}$ we have

$${}_4(a_\mu \gamma^b_\mu)_3 = {}_4(\gamma^b_\mu)_3\, a_\mu$$

For generation of A

$$\Box^2 A_\mu = j_\mu = {}_2(\gamma^a_\mu)_1\, e^{-iq\cdot x}$$
$$q^2 a_\mu = {}_2(\gamma^a_\mu)_1$$
$$a_\mu = {}_2(\gamma^a_\mu)_1\,\frac{1}{q^2}$$

or ${}_4(\gamma^b_\mu)_{32}(\gamma^a)_1 \frac{1}{q^2}$ as above.

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<u>Meson Theory</u>

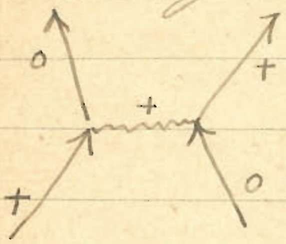
The only way in which theorists have succeeded in representing the interaction between particles in a relativistically invariant manner is by means of a particle exchanged in the interaction. In E.M. theory the photon is the particle exchanged in the interaction between electrons. Mesons theories are developed in analogy with E.M. Theory.

<u>Qualitative aspects of Meson Theory</u>

1) Limited range of nuclear forces ∴ mesons have rest mass.

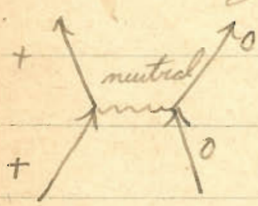
2) Exchange character of nuclear forces.

$\therefore$ charged mesons

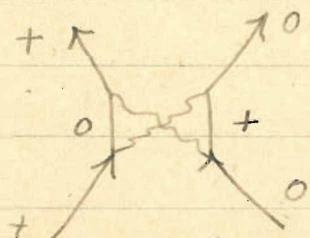


Berkley Exps. verify

3) $p - n \overset{?}{=} p - p \overset{?}{=} n - n$ $\therefore$ neutral mesons or if charged mesons then a large coupling constant $g^2/\hbar c \geq 1$ which makes an exchange of two particles likely



order g



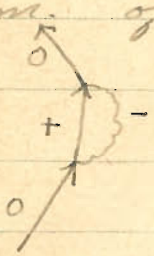
order g^2

The <u>Symmetric Theory</u> uses both charged & neutral mesons which constants adjusted to give

$$p - n = p - p = n - n$$

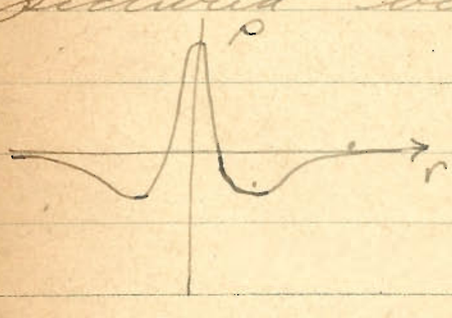
and $m_{neutral} = m_{charged}$

4) Charged mesons will account qualitatively for the mag. moment of the neutron and the anomolous m.m. of the proton.

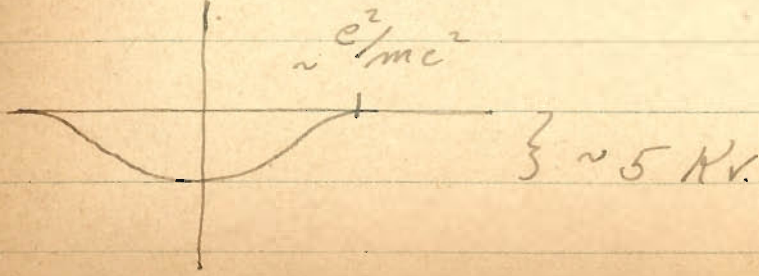


There is no quantitative agreement yet but large + small g theories bracket the correct value.

As a result of the virtual process pictured above the neutron should have a charge distribution as pictured below:

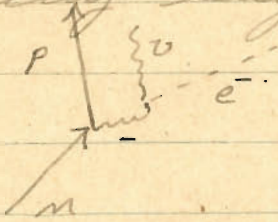


This will generate a potential



5) Spin Dependence of nuclear forces.

6) β decay suggests meson decay



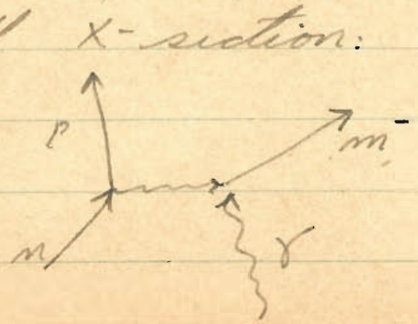
∴ meson unstable

7) "Tensor" forces in deuteron eliminate some theories.

<u>The</u> <u>Theory</u> <u>predicts</u>:

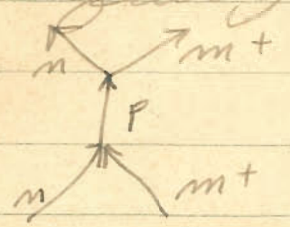
1) Mesons easily produced in High energy collisions

2) Mesons produced by γ rays with small X-section:



3) Mesons easily captured by nuclei ($M > 1$) giving KE to nucleons — stars.

4) Mesons scattered by nucleons with fairly large X-section



It is observed:

1) Mesons in cosmic rays $\mu = 200 \mu_e$

a) $\mu \rightarrow e + (2\nu?)$

but to long lifetime for β decay.

b) Easily produced
c) Not easily captured } Paradox

d) Small scattering with n

e) Yield no stars

f) Spin probably ½

2) Scattering Experiments have been explained on basis of symmetric theory.

3) π - meson. $300\,\mu_e$ $\tau = .7 \times 10^{-8}$ sec
a) No β decay yet observed
b) Easily produced
c) Easily captured - make stars
d) No data on scattering
e) $\pi \rightarrow \mu + (\nu?)$

Classification of Meson Theories

I Neutral vector mesons of zero mass with vector coupling.

Made in strict analogy with QED
Nucleon $\rightarrow$ electron
Meson $\rightarrow$ photon
$g \rightarrow e$
$B_\mu \rightarrow A_\mu$

14 $\Box^2 B_\alpha = j_\mu$

In all meson theories the nucleons are assumed to obey the Dirac Equation

$$(i\not\nabla - M)\psi_{\substack{n\\p}} = (\tfrac{g}{c}\not B + \tfrac{e}{c}\not A)\psi_{\substack{n\\p}}$$

If neutron & proton are fundamental particles this implies that neg protons and anti neutrons should be found. If the evidence that there are no negative protons is sufficiently good maybe one can conclude that a proton is not a fundamental particle.

Interaction

$$\ldots\; g^2\,{}_4(\gamma_\mu)_3\,\frac{1}{q^2}\,{}_2(\gamma_\mu)_1 \qquad q = p_2 - p_1 = p_4 - p_3$$

For slow nucleons

$$q_4 = E_2 - E_1 = \sqrt{M^2+p_2^2} - \sqrt{M^2+p_1^2}$$
$$= \frac{p_2^2}{2M} - \frac{p_1^2}{2M}$$

$$\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1$$

Since $\dfrac{p^4}{M^2} \ll p^2$

$$q_4^2 \ll \mathbf{q}\cdot\mathbf{q}$$

The <u>static</u> <u>approx</u> is $q_4 \simeq 0$ and yields

$$-\,{}_4(\gamma_\mu)_3\,\frac{1}{\mathbf{q}^2}\,{}_2(\gamma_\mu)_1$$

In non-rel. limit

$${}_2(\gamma_4)_1 \to {}_2(1)_1$$

$${}_2(\gamma_x)_1 \to {}_2\left(\frac{p_{1x}+p_{2x}}{2Mc} \mp i\,\frac{\sigma\times\mathbf{q}}{2Mc}\right)_1 \sim 0$$

where the matrix element are taken between the large components.

(For the record

$${}_2(\gamma_x\gamma_y)_1 = i\,{}_2(\sigma_z)_1$$
$${}_2(\gamma_4\gamma_x)_1 = {}_2\left(\frac{\mathbf{q}_x}{2Mc} - i\,\frac{\sigma\times(p_1+p_2)_x}{2Mc}\right)$$
$${}_2(\gamma_5)_1 = i\,{}_2\left(\frac{\sigma\cdot\mathbf{q}}{2Mc}\right)_1$$

)

Our interaction becomes

$$\sim g^2\,{}_4(1)_3\,\frac{1}{\mathbf{q}^2}\,{}_2(1)_1$$

$\therefore$ No spin effects
No exchange

$$\psi(r) = \int \phi(p) e^{i p \cdot r} dp$$

$$_2(1)_1 = \int \phi_2^* \, 1 \, \phi_1 \, dp_1$$

$$= \int \psi_2^* e^{i q \cdot r} \psi_1 \, dr$$

In terms of space variables

$$\sim g^2 \, {}_4(e^{(q \cdot r_B)})_3 \frac{1}{q^2} {}_2(e^{i q \cdot r_A})_1$$

$$\sim g^2 \iiint \psi_{4B}^* \psi_{2A}^* e^{i q \cdot r_{AB}} \frac{dq}{q^2} \psi_{3B} \psi_{1A} \, dr_A \, dr_B$$

Integration over dq accomplished by change to polar coord yields: $\sim \frac{\pi}{r_{AB}}$

This is Rutherford Law.

II Neutral vector mesons of mass m

$$\Box^2 B_\mu - m^2 B_\mu = j_\mu$$ KG equation if $j_\mu = 0$

Continuity equation $\frac{\partial j_\mu}{\partial x_\mu} = 0$ implies

$$\frac{\partial B_\mu}{\partial x_\mu} = 0$$

$\therefore B_\mu$ can be specified by 3 components which transform like a vector. This is characteristic of spin 1.

For a free meson, i.e. no nucleons nearby

$$\Box^2 B_\mu - m^2 B_\mu = 0$$

For $B_\mu = b_\mu e^{iq\cdot x}$

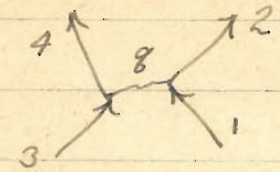
$$(q^2 - m^2) b_\mu = 0$$

$$\frac{\partial B_\mu}{\partial x} \Rightarrow q_\mu b_\mu = 0$$

For a meson at rest $q = 0$, $q_4 = m$

$$q_4 b_4 = 0$$

$$\therefore b_4 = 0$$

Interaction 

$$\sim g^2 \; {}_4\left(\gamma_\mu - \frac{q_\mu q}{m^2}\right)_3 \frac{1}{q^2 - m^2} \; {}_2\left(\gamma_\mu - \frac{q_\mu q}{m^2}\right)_1$$

In non-rel. approx.

$$q_4 \sim 0$$

$${}_2(\gamma_{123})_1 \sim 0$$

$${}_2(\gamma_4)_1 \sim {}_2(1)_1$$

$$-g^2 \, {}_4(1)_3 \frac{1}{q^2 + m^2} \, {}_2(1)_1$$

In real space:

$$\int e^{i q \cdot \underline{R}_{AB}} \frac{1}{m^2 + q^2} \, dq$$

$$= g^2 \frac{e^{-m|R_{AB}|}}{|R_{AB}|} \qquad \text{Yukawa potential}$$

Particles having mass give rise to short range forces. One way of looking at this is to note that since the min. energy associated with virtual emission is the rest energy - which is large - the particle can not stay virtual very long and hence does not go very far.

In the non-rel. approx. m may be a binding energy for the mesons. There are lots of theories involving potential wells but these can not be made rel. invar. unless the well is infinitesimally narrow.

The range of the forces determines

$$\left.\begin{array}{l} m \approx 326\, m_e \\ m_{exp} \approx 280\, m_e \end{array}\right\} \text{not bad}$$

V-Depth determines $g^2/\hbar c \sim .2$

III Charged vector mesons with vector coupling.

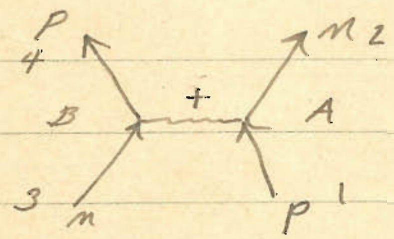
In order to keep track of the charge we invent an operator called the isotopic spin, τ.

Let:

$$\psi_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$\psi_n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \; ; \; \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau_+ \psi_n = \psi_p \; ; \; \tau_+ \psi_p = 0$$
$$\tau_- \psi_n = 0 \; ; \; \tau_- \psi_p = \psi_n$$

Interaction 

$${}_4(\tau_+^B \gamma_\mu^B)_3 \, \frac{1}{q^2 - m^2} \, {}_2(\tau_-^A \gamma_\mu^A)_1$$

$$+ \, {}_4(\tau_-^B \gamma_\mu^B)_3 \, \frac{1}{q^2 - m^2} \, {}_2(\tau_+^A \gamma_\mu^A)_1$$

$$= \frac{1}{q^2 - m^2} \left(\tau_+^B \tau_-^A + \tau_-^B \tau_+^A \right) \gamma_\mu^B \gamma_\mu^A$$

Define: $\tau_1 = \tau_+ + \tau_- = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\tau_2 = -i(\tau_+ - \tau_-) = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Then:

$$\tau_+^B \tau_-^A + \tau_+^A \tau_-^B$$
$$= \frac{1}{2}\left[\tau_1^B \tau_1^A + \tau_2^B \tau_2^A \right]$$

$$\text{Interaction} = \sum_{i=1}^{2} (\tau_i^B \gamma_\mu^B)(\tau_i^A \gamma_\mu^A)$$

τ_1 & τ_2 anticommute. There is a third anticommuting matrix

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\tau_3 \psi_p = \psi_p \ ; \ \tau_3 \psi_n = -\psi_n$$

τ_3 corr to neutral interaction:

<u>Symmetric Theory</u>

$$\frac{1}{q^2 - m^2} \sum_1^3 (\tau_i^B \gamma_\mu^B)(\tau_i^A \gamma_\mu^A)$$

Comparison of interactions

$\tau_1, \tau_2, \tau_3 \to 1, 1, 1.$

	charged	τ_3 neutral	sym.	ordinary neutral
p-p	0	1	1	1
n-n	0	1	1	1
n-p sym.	+2	−1	1	1
anti	−2	−1	−3	

<u>Modification of Interaction</u>

To make up new mesons interactions the γ's may be replaced by what-have-you:

IV Scalar mesons with scalar coupling:

$$\frac{1}{q^2-m^2}\,{}_4(1)_3\,{}_2(1)_1$$

where:

$$\begin{cases} \Box^2 B_1 - m^2 B_1 = {}_2(1)_1 \\ (i\not\nabla - M)\psi = B_1\psi \end{cases}$$

V Scalar mesons with vector coupling:

$$\frac{1}{q^2-m^2}\,{}_4(\not q)_3\,{}_2(\not q)_1$$

where:

$$\begin{cases} \Box^2 B_q - m^2 B_q = {}_2(\not q)_1 \\ (i\not\nabla - M)\psi = B_q\psi \end{cases}$$

This interaction produces no effect:

Proof 1

$${}_2(q)_1 = {}_2(p_2 - p_1)_1 = {}_2(M - M)_1 = 0$$

Proof 2

$$i\not\nabla\psi = (M + \gamma_\mu \frac{\partial\phi}{\partial x_\mu})\psi$$

Let: $\psi = e^{-i\phi}\chi$

$$\not\nabla\psi = \gamma_\mu \frac{\partial\psi}{\partial x_\mu} = \gamma_\mu e^{-i\phi}\frac{\partial\chi}{\partial x_\mu} - \gamma_\mu i \frac{\partial\phi}{\partial x_\mu}\chi e^{-i\phi}$$

$$i\not\nabla\chi = M\chi$$

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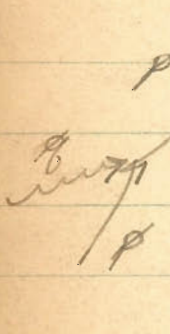
Review

Scattering of a real particle

$$(i\not\nabla - \mu)\psi = \not A\psi$$



After scattering

$$\psi = u e^{-i(p+q)\cdot x}$$

Before

$$= u e^{-ip\cdot x}$$

$$A_\mu = a_\mu e^{-iq\cdot x}$$

To first order:

$$u(\not p + \not q - \mu) = \not a_\mu$$

$$u = \frac{1}{\not p + \not q - \mu}\not a_\mu$$

Propagation of light.

$$\Box^2 A_\mu = j_\mu = c_\mu e^{-iq\cdot x}$$
$$A_\mu = a_\mu e^{-iq\cdot x}$$
$$q^2 a_\mu = c_\mu$$
$$a_\mu = \frac{c_\mu}{q^2}$$

In general case:

$$\int \left(\int \bar\psi_4 \gamma_\mu e^{-iq\cdot x'} \psi_3 dx'\right) \frac{dq}{q^2} \left(\int \bar\psi_2 \gamma_\mu e^{iq\cdot x} \psi_1 dx\right)$$

—— o ——

VI Pseudoscalar mesons with pseudoscalar interaction

$$_4(\gamma_5)_3 \frac{1}{q^2 - m^2} \, _2(\gamma_5)_1$$

Non-rel. approx. $q_4 \sim 0$ $\quad p_2 - p_1 = q$
$M \sim E$

$$_2(\gamma_5)_1 = i\,_2(\sigma_2 \alpha_2)_1 = i\,_2\left(\frac{\sigma\cdot q}{2M}\right)_1 \quad \text{o.k.}$$

$$\ldots \int \left(\frac{\sigma \cdot q}{2M} e^{-iq\cdot R_A} \right)_4 \frac{d^3 q}{q^2+m^2} \left(\frac{\sigma \cdot q}{2M} e^{iq \cdot R_B} \right)_{3\,2\,1}$$

$$= \ldots \int \left(\frac{\sigma \cdot \nabla_A}{2M} e^{-iq\cdot R_A} \right)_4 \frac{dq}{q^2+m^2} \left(\frac{\sigma \cdot \nabla_B}{2M} e^{iq\cdot R_B} \right)_{3\,2\,1}$$

$$= \ldots \frac{1}{(2M)^2} {}_4(\sigma \cdot \nabla_A)_{32} (\sigma \cdot \nabla_B)_1 \frac{e^{-m r_{AB}}}{r_{AB}}$$

$$= \frac{g_{ps}^2}{(2M)^2} m^2 \Big[\frac{1}{3} S_{AB} \left(\frac{3}{m^2 r^3} + \frac{3}{m r^2} + \frac{1}{r} \right) e^{-mr}$$

$$+ \frac{1}{3} (\sigma_A \cdot \sigma_B) \frac{1}{r} e^{-mr} \Big]$$

where $S_{AB} = 3\left(\sigma_A \cdot \frac{r}{|r|}\right)\left(\sigma_B \cdot \frac{r}{|r|}\right) - (\sigma_A \cdot \sigma_B)$

S_{AB} is the tensor force. It varies in space like $P_2 = \frac{3}{2}\cos^2 - \frac{1}{2}$

Note that the $\frac{1}{r^3}$ terms means collapse of nucleus since $KE \sim \frac{1}{r^2}$. Artificial "cut-offs" attempt to save the situation

Phenomenalogical theory which includes tensor forces:

$$S_{AB}\, V(r) + \sigma_A \cdot \sigma_B\, V'(r)$$

where for $V(r)$ & $V'(r)$ one can insert for example $\frac{1}{r}e^{-mr}$

On the basis of this theory

$$\frac{g_{PS}^2}{\hbar c}\left(\frac{m}{M}\right)^2 = .2$$

$$\frac{g_{PS}^2}{\hbar c} \sim 40$$

The coupling is very large. It is therefore incorrect to use the 1^{st} order result. There is small hope that calc. the coupling in second order will lead to a smaller value.

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VII Pseudoscalar mesons with pseudovector coupling

$$\frac{g_v^2}{m^2}\,{}_4(\gamma_5 q)_3\,\frac{1}{q^2-m^2}\,{}_2(\gamma_5 q)_1$$

$${}_2(\gamma_5 q)_1 = {}_2(\gamma_5 p_2 - p_1)_1 = {}_2(-p_2\gamma_5 - \gamma_5 p_1)_1$$

$$= -2M\,{}_2(\gamma_5)_1$$

$$g_v^2\left(\frac{2M}{m}\right)^2 {}_4(\gamma_5)_3\,\frac{1}{q^2-m^2}\,{}_2(\gamma_5)_1$$

$\therefore$ 1st order: pvc = psc

$$g_v^2 = \left(\frac{m}{2M}\right)^2 g_{ps} = .2\hbar c$$

In 2nd order pvc $\neq$ psc

	1st	2nd
psc	$(\gamma_5) \sim q$	$(\gamma_5)^2 \sim 1$
pvc	$(\gamma_5 q) \sim q$	$(\gamma_5 q)^2 \sim q^2$

VIII Vector mesons with tensor coupling

$$\frac{1}{4}(\gamma_\mu q - q\gamma_\mu)_3 \frac{1}{q^2-m^2} \frac{1}{2}(\gamma_\mu q - q\gamma_\mu)_1$$

In non-rel. approx

$$\gamma_\mu q \sim (\sigma \cdot q)$$

$\therefore$ same as ps to this order, but sign of tensor force is reversed giving wrong sign to quad. moment of deutron.

In an attempt to straighten out the various difficulties of the theories combinations have been tried.

Moller-Rosenfeld: psc & pcv combined to eliminate tensor term

Moller Rosenfeld Schwinger: ps & V mesons with T interaction combined and mass adjusted to get rid of tensor terms.

Relativity has an effect which is not clear.

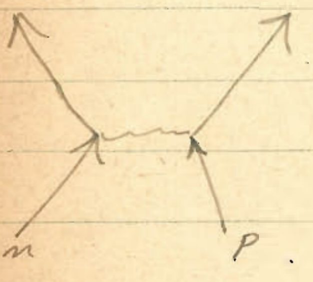
Strong Coupling

$$\frac{g^2}{\hbar c} \to \infty = \hbar \to 0$$

This csp to the classical limit. A classical theory with spin has been worked out. One consequence is excited states of nucleons.
S.C. is probably not correct.

N-P Scattering

with PSC : neutral

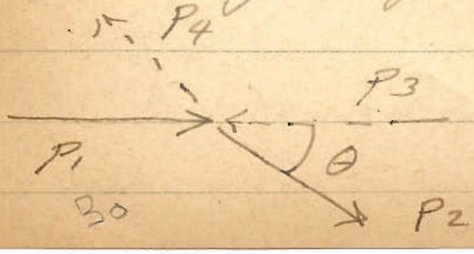


P.S.M.

$$(\gamma_5)_{43} \frac{1}{q^2-m^2} (\gamma_5)_{21}$$

In c.g. system

$$q = p_2 - p_1$$
$$q_4 = 0$$



$$|q| = 2p \sin \theta/2$$

$$E_{nucleons} = \sqrt{M^2+p^2}$$

$$\frac{1}{4p^2\sin^2\frac{\theta}{2}+m^2}\ {}_4(\gamma_5)_{32}(\gamma_5)_1$$

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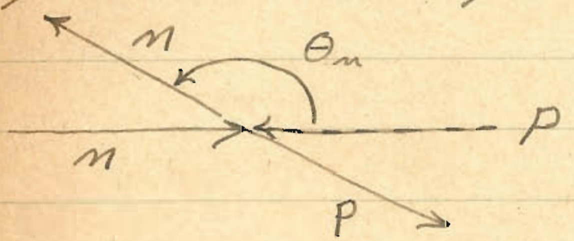
N-P Scattering

psm with psc : charged



$$\frac{1}{4p^2\cos\frac{\theta_n}{2}+m^2}\ {}_4(\sigma\cdot q)_{32}(\sigma\cdot q)_1$$

Note on Spin Sums

$${}_2(M)_1 = \tilde{u}_2 M u_1$$

Prob $\xrightarrow{M} 2 = |{}_2(M)_1|^2$

In general an experiment averages over the spins:

$$\frac{1}{2}\sum_{\substack{spin\\2}}\sum_{\substack{spin\\1}} |{}_2(M)_1|^2$$

$$\sum_{1} \left(\frac{\tilde{u}_2}{\|u_2\|} M \frac{u_1}{\|u_1\|}\right)\left(\frac{\tilde{u}_1}{\|u_1\|} \tilde{M} \frac{u_2}{\|u_2\|}\right)$$

In performing the sum over spin states we want to include only positive energy states yet we would like to be able to sum over the complete set of u's. This may be accomplished by means of projection operators

$$\sum_{1} \left(\frac{\tilde{u}_2}{\|u_2\|} M \frac{(p_1 + \mu)}{2\mu} \frac{u_1}{\|u_1\|}\right)\left(\frac{\tilde{u}_1}{\|u_1\|} \tilde{M} \frac{u_2}{\|u_2\|}\right)$$

$$= \frac{1}{2\mu}\left(\frac{\tilde{u}_2}{\|u_2\|} M (p_1 + \mu) \tilde{M} \frac{u_2}{\|u_2\|}\right)$$

$$\frac{1}{2} \sum_{2} \frac{1}{2\mu}\left(\frac{\tilde{u}_2}{\|u_2\|} M (p_1 + \mu) \tilde{M} \frac{(p_2 + \mu)}{2\mu} \frac{u_2}{\|u_2\|}\right)$$

$$= \frac{1}{2\mu^2} \text{ Spur } \left(M (p_1 + \mu) \tilde{M} (p_2 + \mu)\right)$$

Where $\text{Spur}(1) = \frac{1}{4} \sum_{i=1}^{4} \frac{(\tilde{u}_i u_i)}{(\tilde{u}_i u_i)} = 1$

$$Sp(\gamma_\mu) = Sp(\gamma_\mu \gamma_\nu) = Sp(\gamma_\mu \gamma_\nu \gamma_\sigma) = Sp(\gamma_5) = 0$$

Lemma Spur (AB) = Spur (BA)

$$Sp(\not p_1 \not p_2) = -Sp(\not p_2 \not p_1) + 2Sp(p_1 \cdot p_2)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad \overset{\shortparallel}{2p_1 \cdot p_2}$$

$$Sp(\not p_1 \not p_2) = p_1 \cdot p_2$$

et cetera

$$\frac{1}{2}\sum |_2(M)_1|^2 = \frac{1}{2\mu^2}(\tilde{u}_2 u_2)(\tilde{u}_1 u_1) Sp \ldots$$

$$= \frac{1}{2}\frac{1}{E_2 E_1} Sp\{M(\not p_1 + \mu)\tilde{M}(\not p_2 + \mu)\}$$

For our problem: $M = \gamma_{5-} = \gamma_x \gamma_y \gamma_z \gamma_t$
$$\tilde{M} = -\gamma_t \gamma_z \gamma_y \gamma_x = -\gamma_{5-}$$

$$Sp[(\not p_2 + M)\gamma_{5-}(\not p_1 + M)\gamma_5] Sp[(\not p_4 + M)\gamma_5(\not p_3 + M)\gamma_5]$$

$$= (p_1 \cdot p_2 - M^2)(p_3 \cdot p_4 - M^2)$$

Note: $q = p_2 - p_1 = p_4 - p_3$
$$q^2 = p_2^2 - 2p_1 \cdot p_2 + p_1^2 = 2M^2 - 2p_1 \cdot p_2$$
$$= 2M^2 - 2p_3 \cdot p_4$$

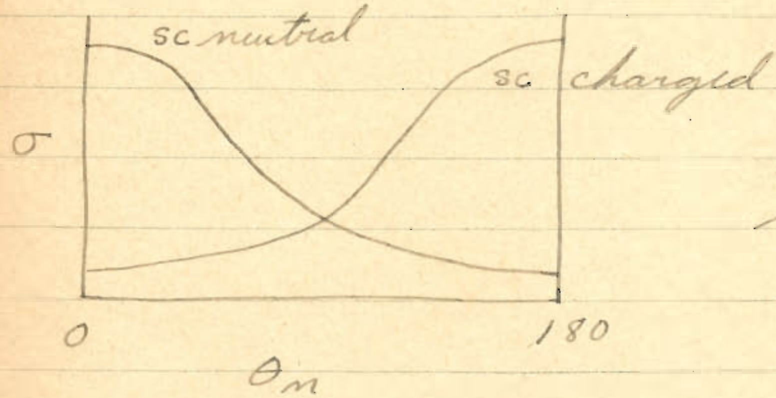
X-section in C.G. system

$$\sim \left(\frac{q^2}{q^2-m^2}\right)^2$$

$$\sim \left(\frac{4p^2\cos^2\frac{\theta}{2}}{4p^2\cos^2\frac{\theta}{2}+m^2}\right)^2$$

Compare with X-section for neutral theory

$$\sim \left(\frac{1}{4p^2\sin^2\frac{\theta}{2}+m^2}\right)^2$$



In c.g. system

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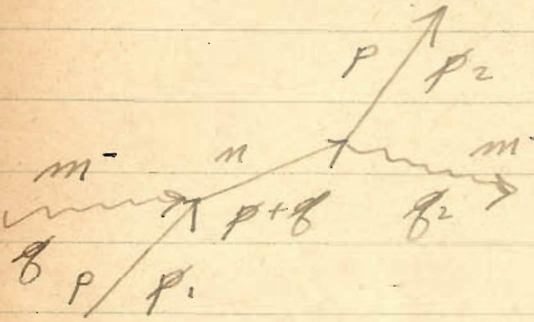
Analogue of Klein Nishina formula for mesons: Scattering of mesons

PSM – PSC – charged



$q^2 = m^2$
$p^2 = M^2$
$p_1 + q_1 = p_2 + q_2$

$$= \left(\gamma_5 \frac{1}{p_1 + q_1 - M} \gamma_5 \right)_{1} \quad = (q_1)_1 / (m^2 + 2 p_1 \cdot q_1)$$

$$\sigma \sim (m^2 + 2 p_1 \cdot q_1)^{-2} Sp[(p_2 + M) q_1 (p_1 + M) q_1]$$

$$= (m^2 + 2 p_1 \cdot q_1)^{-2} [2 (p_2 \cdot q_1)(p_1 \cdot q_1) - (p_1 \cdot p_2 - M^2) q_1^2]$$

$$\begin{cases} (p_1 - q_2)^2 = (p_2 - q_1)^2 \\ p_1 \cdot q_2 = p_2 \cdot q_1 \end{cases}$$

$$\begin{cases} (p_1 - p_2)^2 = (q_2 - q_1)^2 \\ M^2 - 2 p_1 \cdot p_2 = m^2 - q_1 \cdot q_2 \end{cases}$$

$$p_1 \cdot (p_1 - p_2) = p_1 \cdot (q_2 - q_1) = p_1 \cdot q_2 - p_1 \cdot q_1$$

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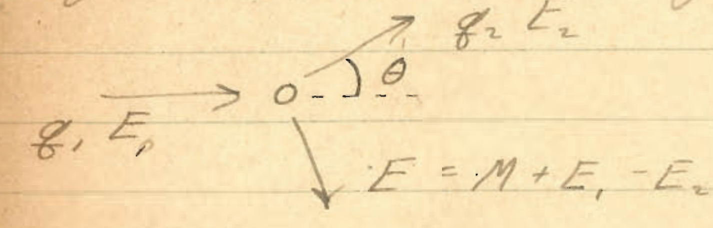
$= (m^2 + 2p_1 \cdot q_1)^{-2} [2(p_1 \cdot q_2)(p_1 \cdot q_1) - (p_1 \cdot q_1 - p_1 \cdot q_2) q_1^2]$

If nucleon initially at rest



$p_1 = \gamma_4 M$

X-sec. $\sim [2E_2E_1 - (E_1 - E_2)\frac{m^2}{M}] / [2E_1 + \frac{m^2}{M}]^2$

$$M^2 - M(M + E_1 - E_2) = m^2 - [E_1E_2 - |q_1||q_2|\cos\theta]$$

$$|q_1||q_2|\cos\theta = E_1E_2 - m^2 - M(E_2 - E_1)$$

Neglect $\frac{m}{M}$ terms

$$\text{X-sec}/_{d\Omega} = 2.6 \times 10^{-28} \left(\frac{g^2}{\hbar c}\right)^2 \left(\frac{E_2}{E_1}\right)^2 \cdot \left(\frac{E_2}{E_1}\right) \text{ cm}^2$$

Problem: Do Scalar Case

Ans:

$$\delta p = \frac{\left(8M^2 + 8E_1M + 2E_1E_2 + (E_2 - E_1)\frac{m^2}{M}\right)}{\left(2E_1 + \frac{m^2}{M}\right)^2}$$

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Relativity

Lorentz Transformation: relative motion in x-direction, relative velocity β.

$$E' = \gamma(E \pm \beta p_x) \quad : \gamma = (1-\beta^2)^{-1/2}$$

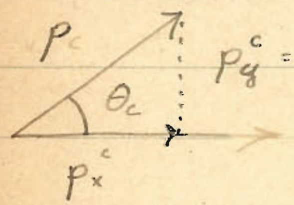
$$p_x' = \gamma(p_x \pm \beta E)$$

$$p_y' = p_y$$

$$p_z' = p_z$$

If particle is going slower in ' system than in unprimed system then minus sign is to be used and vice versa

Angle transformation



$$\tan\theta_L = \frac{p_y^L}{p_x^L} = \frac{p_y^c}{\gamma(p_x^c + \beta E^c)} = \frac{p^c \sin\theta_c}{\gamma(p^c\cos\theta_c + \beta E^c)}$$

Transformation to C.M. system

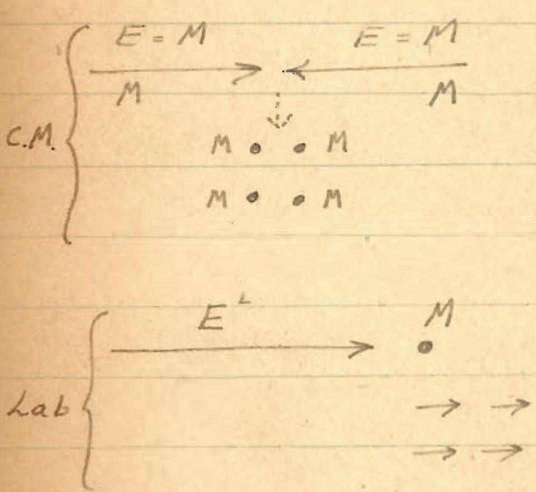
In CM system $\sum p^c = 0$

$$0 = \sum p^c = \gamma\left(\sum p^L - \beta \sum E^L\right)$$

$$\boxed{\beta = \sum p^L \Big/ \sum E^L}$$

$$\boxed{\left(\sum E^c\right)^2 = \left(\sum E^L\right)^2 - \left(\sum p^L\right)^2}$$

Nucleon Pair Creation:



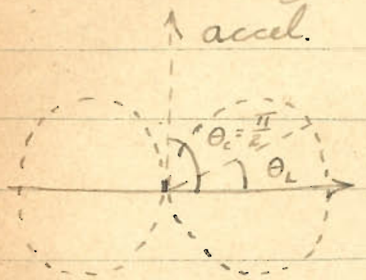
$$(4M)^2 = (E^L+M)^2 - (E^{L2}-M^2)$$

$$16M^2 = \cancel{E^{L2}} + 2E^L M + M^2 - \cancel{E^{L2}} + M^2$$

$$E^L = 7M$$

$$E^L_{KIN} = 6M$$

Radiation from Synchrotron Orbit



$$\theta_c = \frac{\pi}{2}$$

$$\theta_L = \frac{p_c'}{\gamma\beta E_c'}$$

$$\frac{1}{\gamma} = \sqrt{1-\beta^2} \doteq \sqrt{1-\left(\frac{p^L}{E^L}\right)_e^2} = \frac{m_e}{E_e^L}$$

' = photon $\qquad |p_c'| = E_c'$

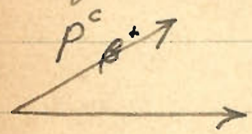
$$\beta \approx 1$$

$$\theta_L \doteq \frac{m}{E_e^L} = \frac{1}{600} \text{ radians}$$

Electron-Electron Collision: Energy in C.M. system if one elec. at rest in lab

$$\frac{E_c}{m_e} = \sqrt{2\left(\frac{E_l}{m_e}+1\right)}$$

Angular Distribution: β^+'s from $\mu^+ \to e^+ + \nu$



$$n(E_L)\,dE_L = n(\theta_c)\,d\theta_c$$

$$n(E_L) = n(\theta_c)\left|\frac{d\theta_c}{dE_L}\right|$$

$$E_L = \frac{E_c + \beta p_x^c}{\sqrt{1-\beta^2}} = \frac{E_c + p_x^c}{\left(\frac{\mu}{E_L}\right)_{meson}} = \frac{p^c(1+\cos\theta_c)}{\left(\frac{\mu}{E_L}\right)_{meson}}$$

$$\left|\frac{dE_L}{d\theta_c}\right| = p_c \sin\theta_c \Big/ \left(\frac{\mu}{E_L}\right)_{meson}$$

If β^+'s emitted isotropically in C.M. system:

$$n(\theta_c)\,d\theta_c = 2\pi \sin\theta_c \, d\theta_c$$

$$n(E_L) = 2\pi \sin\theta_c \left(\frac{\mu}{E_L}\right)_{meson} \Big/ p_c \sin\theta_c$$

$$= \frac{2\pi}{p_c}\left(\frac{\mu}{E_L}\right)_{meson} \quad : \; \beta_c \cong 1$$

__A Useful Trick__

If an electron goes by a heavy nucleus, it interacts strongly with the coulomb field over a distance ~2 times impact parameter. If we replace the interaction by a wave, $2b=\lambda$, we have a collision between an electron and a soft quantum. Since the nucleus is so heavy transforming to C.M. system will not much affect the energy of the quantum.

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Capture of π-Mesons

A π^- gets caught in an atomic orbit. It drops down to the K shell by giving its energy to atomic electrons. It then sits by nucleus until captured:

For this process:

$$p_2 = p_1 + q$$

Here the state 2 is virtual since E & p cannot be conserved in the process shown. However there are plenty of nucleons around to collide with 2 and make the process real.

For arbitrary wave functions

$$M_{fi} = \int \bar{\Psi}_f^n e^{iqx} \gamma_5 \Psi_i^p \, dx : \text{P-Scalar } C$$

$$M_{fi}^{\substack{plane\\wave}} = g \int \bar{\Psi}_f \left(\sum_i \gamma_5^j \tau_-^j e^{iq_\mu x_\mu^j} \right) \Psi_i \prod_j dx^j$$

For a meson in an orbit

$$\Psi_{i(x)}^m = \int u(q) e^{iq\cdot\underline{x}} dq$$

Integrate integral for M_{fi}^{pw} over dt

$$M_{fi}^{pw} = g \int \bar{\Psi}_f \left(\sum_i \gamma_5^j \tau_-^j e^{iq\cdot\underline{x}^j} \right) \Psi_i \prod_j d\underline{x}^j$$

where now $E_f = E_i + q_4$.

$$M_{fi}^{orbit} = g \int M^{pw} u(q) \, dq$$

$$= g \int \bar{\Psi}_f \left(\sum \gamma_5^j \tau_-^j \Psi_{(\underline{x}_j)}^m \right) \Psi_i \prod_j d\underline{x}^j$$

I Over the region where Ψ_i appreciable $\Psi_{(x^j)}^m \sim \text{constant} = \Phi(0)$

Prob capture / sec = 1 / Lifetime

$$= \frac{2\pi}{\hbar} g^2 \left| \int \Psi_f \left(\sum_j \gamma_5^j \tau_-^j \right) \Psi_i \, dV \right|^2 \rho(E_f) |\Psi^m(0)|^2$$

Calculation of $\Psi^m(0)$. Assume meson satisfies non-relativistic Schroedinger Equation.

$$\varphi^m(r) = \varphi(0) e^{-\alpha r}$$

$$\alpha = Z m e^2 / \hbar^2 \qquad m = \text{meson mass}$$

From normalization $\int |\varphi(r)|^2 r^2 dr = \frac{1}{4\pi}$

$$|\varphi(0)|^2 = 4\alpha^3 = 4 Z^3 \left(\frac{m}{\mu_e}\right)^3 a_0^{-3}$$

For $Z = 10$

$$\frac{1}{\alpha} = \frac{a_0 \mu_e}{Z m} = \frac{1}{3000} \overset{.5 \times 10^{-8}}{a_0} = 1.7 \times 10^{-12} \text{ cm}$$

Hence meson is inside nucleus most of the time.

Estimation of $\left| \int \Psi_f (\Sigma ..) \Psi_i \, dV \right|^2$ is difficult. One can get some idea of its value by noting that if we call:

$$\left| \int \Psi_f \tau_- \Psi_i \, dv \right|^2 \equiv M(E)$$

then:

$$\int \rho(E) M(E) \, dE = 1$$

$$\int \rho(E) M(E) E \, dE$$ = diff in energy of original nucleus and nucleus with one proton changed to a neutron.

?

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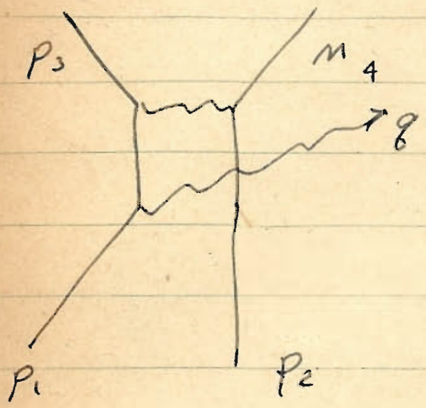
Production of Mesons by Nuclear collisions



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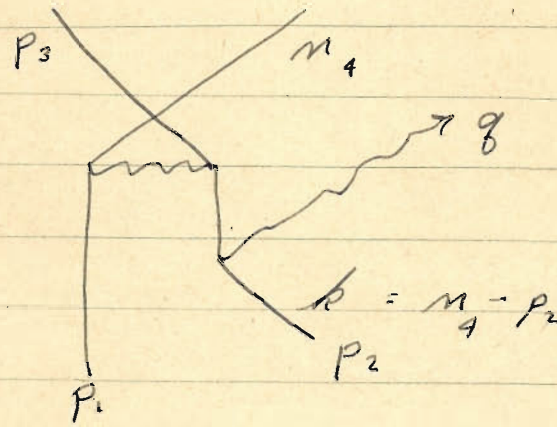


II

Scalar Theory

$$M_{fi} = g^3\left\{\frac{1}{k^2-m^2}\,{}_4(1)_2\left(1\,\frac{1}{p_1 - q - M}\,1\right)_1\right.$$

$$\left. - \frac{1}{k^2-m^2}\,{}_4(1)_3\left(1\,\frac{1}{p_2 - q - M}\,1\right)_2\right\}$$

For non relativistic nucleons

$$\left(\frac{1}{p - q - M}\right) = \frac{(2M - q)}{-2p\cdot q + q^2} \simeq -\frac{(1)}{q_4 - \frac{p\cdot q}{M}} \simeq -\frac{(1)}{q_4}$$

$$M_{fi} = g^3\left\{{}_2(1)_4\frac{1}{(p_2 - M_4)^2 - m^2}\,{}_3(1)_1 - {}_4(1)_1\frac{1}{(p_1 - M_4)^2 - m^2}\,{}_3(1)_2\right\}\frac{1}{q_4}$$

$\{\ \}$ is just the expression for scattering of nucleons. If this is obtained from experiment then emission rate may be obtained by multiplication by emission constant $(\cdot \frac{1}{q_4})$ given by theory.

The conventional way of treating this problem is to calc.

$$M_{fi} = g^3 \sum_k \frac{\int \Psi_3^{*n}(1)\, \Psi_4^{*p}(2)\, V(r_{12})\, \Psi_k^{n}(1)\, \Psi_2^{p}(2)\, d\tau_1 d\tau_2}{E_k + q_4 - E_1} \times \int \Psi_k(r)\, e^{i q \cdot r}\, \Psi_1(r)\, dr$$

$$- \quad \cdots$$

$$= g^3 \sum_k \frac{{}_{43}(V(12))_{k2}\; {}_k(1)_1}{E_k + q_4 - E_1} \quad - \quad \cdots$$

For plane waves:

$$p_k = p_1 - q_2$$

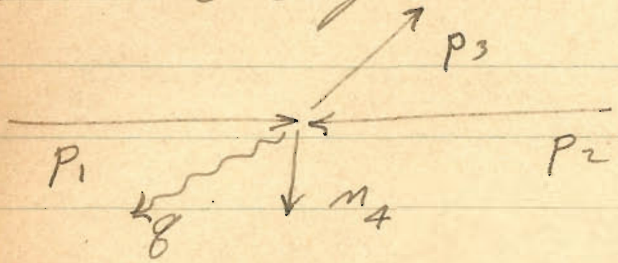
$$E_k - E_1 = \frac{p_k^2 - p_1^2}{2M}$$

$$\doteq p_1 \cdot q / 2M$$

Hence:

$$M_{fi} \doteq g^3 \left\{ {}_{43}V_{21} - {}_{43}V_{12} \right\} \frac{1}{q_4}$$

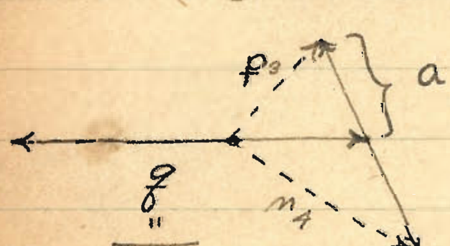
In C G system:



$$\# / \text{sec} = \frac{2\pi}{\hbar^3 c^2} \left| V(p_0) \right|^2 g^2 \frac{1}{q_4^2} \frac{dN_f}{dE_f} \cdot \frac{1}{m}$$

(need dimensionally)

Near threshold $q_4 \sim m$



$q = \sqrt{2m\epsilon}$

Available Energy $= \epsilon_0 = 2\frac{p_1^2}{2M} - m$

Meson energy $= \epsilon \ll \epsilon_0$

$$\sqrt{2M(\epsilon_0 - \epsilon)} \gg \sqrt{2m\epsilon} = q$$

Hence we can neglect q in calculating a.

$$p_3 \approx a \approx \sqrt{2M\left(\frac{\epsilon_0 - \epsilon}{2}\right)}$$

$$\frac{dN}{dE} = \frac{(4\pi)^2 p_3^2\, dp_3\, n_4^2\, dn_4}{(2\pi\hbar)^6\, dE} = \frac{(4\pi)^2 p_3^2\, dp_3\, q^2\, dq}{(2\pi\hbar)^3\, dE}$$

$$\frac{dP}{dE} = \frac{M}{P} \; ; \quad Q\,dQ = m\,d\epsilon$$

$$\frac{dN}{dE} = (4\pi)^2 P\, M\, Q\, m\, d\epsilon$$

$$\underset{\text{prod.}}{\text{x-sec}} \sim \frac{2\pi}{\hbar} |V|^2 \frac{g^2}{\hbar^2 c^2} \frac{M^{3/2} (\epsilon_0 - \epsilon)^{\frac{1}{2}} \epsilon^{\frac{1}{2}} d\epsilon\, (4\pi)^2}{m^{3/2} (2\pi\hbar)^6}$$

$$\underset{\text{scat}}{\text{x-sec}}\ 90^\circ \sim \frac{2\pi}{\hbar} |V|^2 \frac{\rho M}{(2\pi\hbar)^3} 4\pi$$

$$\text{x-sec prod.} / \text{x-sec scat} \sim g^2 \frac{(\epsilon_0 - \epsilon)^{\frac{1}{2}} \epsilon^{\frac{1}{2}}}{m^2} \leftarrow m^{3/2} \epsilon_0^{1/2}$$

$$\text{total emission} / \text{total scat} \sim g^2 \left(\frac{\epsilon_0}{m}\right)^2$$

<u>Meson Production</u> P.S. Theory

$$M_{fi} = \frac{1}{(p_2 - n_4)^2 - m^2}\, {}_4(\gamma_5)_{23} \left(\gamma_5 \frac{1}{\not p - \not q - M} \gamma_5\right)_1$$

$$- \ldots$$

$$= \frac{1}{(p_2 - n_4)^2 - m^2}\, {}_4(\gamma_5)_{23} (\not q)_1 / 2M q_4 - \ldots$$

For scattering

$$M_{fi} = \frac{1}{(p_2 - n_4)^2 - m^2}\, {}_4(\gamma_5)_{23} (\gamma_5)_1$$

With P.S. theory one can get

emission rate by multiplying scattering x-section by an "emission" constant.

Near threshold: $\quad {}_3(\gamma)_1 = \gamma_4\,{}_3(\gamma_4)_1 \approx m$

$$ {}_3(\gamma_5)_1 = {}_3(\sigma\cdot q/2M)_1 $$

$\therefore\ \frac{1}{|q|}$ instead of $\frac{1}{q_4}$ as in S theory

$$ \frac{\text{x-sec. prod}}{\text{x-sec. scat}} \sim (\epsilon_0-\epsilon)^{1/2}/\epsilon^{1/2} $$

<u>Meson Decay</u>

One would hope to be able to correlate β-decay rates with meson decay rates. At present there is no theory which does this. The general method of approach is as follows:

Assume:

$$ \pi \rightarrow el + \nu $$

Then for β-decay

$$ n \rightarrow p + \pi \rightarrow p + e + \nu $$

Assume that if π present it has an amplitude $G(1)$ (ind.

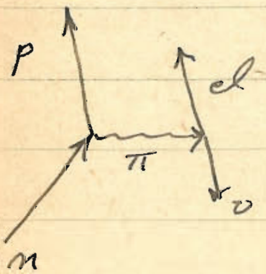
of energy) of being $e + v$.

Prob of disint of free / sec

$$= \frac{1}{{}_{\pi}\tau_{el:v}} = \frac{2\pi}{\hbar}|G|^2\frac{dN}{dE}$$

$$E_{el} \approx E_0 = \tfrac{1}{2} m$$

$$= \frac{1}{8\pi\hbar c}|G|^2\left(\frac{mc^2}{\hbar}\right)$$



Scalar coupling:

$$g\,\frac{(1)}{q^2-m^2}\cdot G \qquad q^2 \ll m^2$$

$$= g\,G/m^2$$

Prob/sec $= \dfrac{2\pi}{\hbar}\,\dfrac{g^2G^2}{m}\,\dfrac{1}{m^4}\,\dfrac{p_e^2\,dp_e\,p_v^2\,dp_v}{(2\pi\hbar)^6\,dE}$

$$= \dots \frac{g^2G^2}{\hbar}\,\frac{p_e\,e_e\,de_e\,e_v^2}{m^5}$$

$$= \dots \frac{g^2G^2}{\hbar}\,\frac{(\epsilon_0-\epsilon_e)^2\sqrt{\epsilon_e^2-\mu_e^2}\,\epsilon_e\,d\epsilon_e}{m^5}$$

Total prob / sec

$$= \dots \frac{g^2}{\hbar c}\,\frac{G^2}{\hbar c}\left(\frac{W}{mc^2}\right)^5\left(\frac{mc^2}{\hbar}\right)$$

$$\frac{1}{\tau_{neut}} = \frac{1}{\tau_\pi}\,\frac{g^2}{\hbar c}\left(\frac{W}{mc^2}\right)^5$$

Morrison says: $\tau_\mu \approx 10^{3}$ sec. ; $\frac{g^2}{\hbar c} = .2$

$\tau_\pi = 3 \times 10^{-11}$ sec.

<u>Photoelectric</u> <u>Effect</u>

Finkelstein

$e^2 = 1/137$

$\hbar = c = 1$

$|k| > I_K$ = ionization energy for K shell.

$$\underline{p} = \underline{k} + \underline{q}$$
$$E = \mu + k - I_K$$

For $k \gg \mu$

$$p = \sqrt{E^2 - \mu^2} = \sqrt{(k + \mu - I)^2 + \mu^2}$$
$$\doteq k + \mu - I - \mu^2/2k$$

$\#$ process / sec $= \frac{2\pi}{\hbar} |H_{fi}|^2 \rho(E)$

For x pol. $H_{fi} = e\sqrt{\frac{2\pi}{k}} \int \psi_f^* \alpha_x e^{i\underline{k}\cdot\underline{r}} \psi_i \, d\underline{r}$

$$\psi_i(\underline{r}) = \int u_i(p) e^{i\underline{p}\cdot\underline{r}} \, d\underline{p}$$

$$\psi_f(\underline{r}) = u_f \, e^{i\underline{p}_f \cdot \underline{r}}$$

$$H_{fi} = \ldots \int u_f^* e^{-i\underline{p}\cdot\underline{r}} \alpha_x e^{i\underline{k}\cdot\underline{r}} \psi_i \, d\underline{r}$$

$H_{fi} = \ldots \; u_f^{*} \alpha_x \, u_i(q)$

For ψ_i we can use the Schrodinger wave function for hydrogen like atoms:

$$\psi_i = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a}\right)^{3/2} e^{-\frac{rZ}{a}}$$

$$u_i(q) = \ldots \left(\frac{Z}{a}\right)^{3/2} \int e^{-\frac{rZ}{a}} e^{i q \cdot r} \, dr$$

$$= \frac{\sqrt{8}}{\pi} \frac{\lambda^{5/2}}{(\lambda^2 + q^2)^2} \quad \text{where } \lambda = \frac{Z}{a} = \sqrt{2\mu I_K}$$

$$|H_{fi}|^2 = \ldots \frac{e^2}{k} \sum_s |u_f^{*} \alpha_x u_i|^2 \frac{\lambda^5}{(\lambda^2 + q^2)^4}$$

Sum over spins

$$\sum_{spin} |u_f^{*} \alpha_x u_i|^2 = \mathrm{Spur}\left(\frac{E - \alpha \cdot p - \beta\mu + 2\alpha_x p_x}{2E}\right)$$

$u_i(q)$ may be computed from the Dirac amplitudes for the H atom given in Rogansky p 527.

One obtains for the matrix elements:

$$ {}_i(1)_i = 1 + \frac{1}{4}\frac{q^2}{\mu^2} $$

$$ {}_i(\alpha)_i = q/\mu $$

$$ {}_i(\beta)_i = 1 - \frac{1}{4}\frac{q^2}{\mu^2} $$

$$ |H_{fi}|^2 = \ldots \frac{e^2}{k}\frac{1}{2\mu(k+\mu)}\left(\frac{k}{4\mu}q^2 + 2q_x^2\right)\frac{\lambda}{(\lambda^2+q^2)^4} $$

Average over x + y polarizations = $\frac{1}{2}$ sum of x + y polarizations

$$ |H_{fi}|^2 = \ldots \left(\frac{k}{4\mu}q^2 + (q_x^2+q_y^2)\right)\ldots $$

In the extreme relativistic limit the polarization terms are negligible

$$ d\phi \overset{rel}{=} \# \times \frac{e^2}{\mu}\frac{k}{\mu}\frac{\lambda^5}{(\mu^2+k^2\theta^2)^3}\, d\Omega $$

we have used $(a^2+q^2) = (\mu^2+k^2\theta^2)$ which is valid for small θ

Total x-section

$$ \phi_d = \# \frac{e^2\lambda^5}{\mu^6 k} = \# \left(\frac{1}{137}\right)\left(\frac{Z}{137}\right)^5\frac{1}{\mu k} $$

The x-section for Thomson scattering

$$\phi_T = \frac{8\pi}{3} r_0^2 = \frac{8\pi}{3}\left(\frac{1}{137}\right)^2 \frac{1}{\mu^2}$$

$$\frac{\phi_{el}}{\phi_T} = \frac{3}{2} \frac{Z^5}{137^4} \frac{\mu}{k}$$: ϕ_{el} = total for K shell.

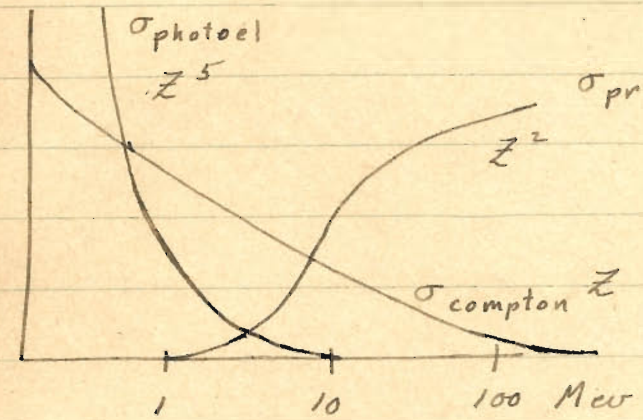
Experimental check: Zatyshev RMP 19
finds Z^n : $n = 4.6 \pm .25$ at 8-9 Mev.

The Z^5 may be understood in the following way:

1) Transition prob. $\sim Z^2$

2) The most important contribution to the photoeffect comes from the region close to the nucleus - say a Compton wave length in radius. Z^3 gives the ratio of the volume of this region to the volume of the K shell and hence is a measure of the time an electron in the K shell spends in the important region

For a graph of $\sigma_{photoel.}$ see Heitler p. 216.

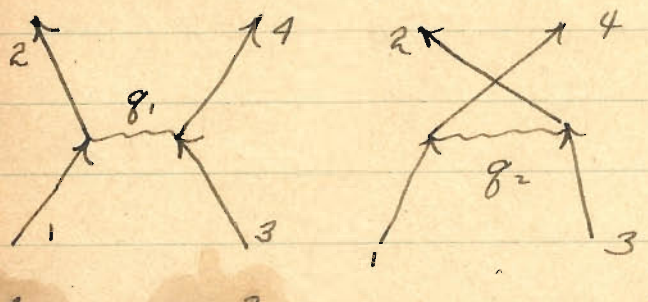


Curves for Pb

Electron - Electron Scattering   Frank

electron - electron : Møller AdP 14 686 1932

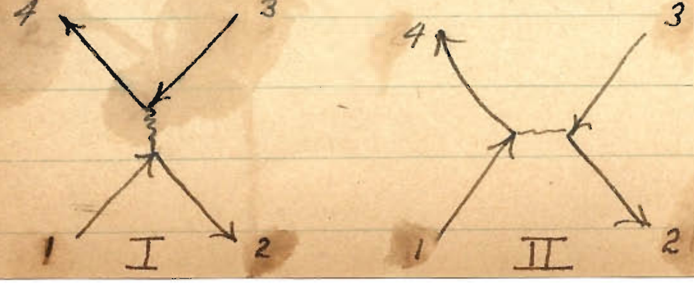
" - positron : Bhabha P.R.S. 154A 195 1936



elec - elec

$q_1 = p_1 - p_2 = p_4 - p_3$

$q_2 = p_1 - p_4 = p_2 - p_3$



elec - pos

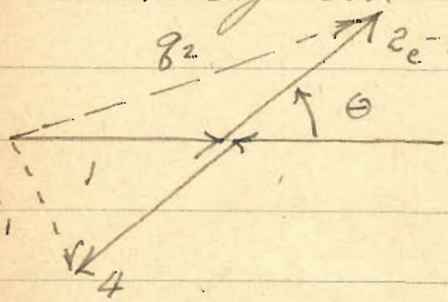
$$M_I = {}_4(\gamma_\mu)_3 \, \frac{1}{q_1^2} \, {}_2(\gamma_\mu)_1$$

$$M_{II} = {}_2(\gamma_\mu)_3 \, \frac{1}{q_2^2} \, {}_4(\gamma_\mu)_1$$

$$|M|^2 = |M_I - M_{II}|^2 = |M_I|^2 + |M_{II}|^2 - \tilde{M}_I M_{II} - \tilde{M}_{II} M_I$$

$$\begin{aligned}
\sum_s |M|^2 = \; & Sp\left[(p_4+\mu)\gamma_\mu(p_3+\mu)\gamma_\mu\right] \\
& \times Sp\left[(p_2+\mu)\gamma_\mu(p_1+\mu)\gamma_\mu\right] \frac{1}{q_1^4} \\
& + Sp\left[(p_2+\mu)\gamma_\mu(p_3+\mu)\gamma_\mu\right] \\
& \times Sp\left[(p_4+\mu)\gamma_\mu(p_1+\mu)\gamma_\mu\right] \frac{1}{q_2^4} \\
& - 2\, Sp\left[(p_4+\mu)\gamma_\mu(p_3+\mu)\gamma_\mu(p_2+\mu)\gamma_\nu(p_1+\mu)\gamma_\nu\right] \\
& \times \frac{1}{q_1^2}\frac{1}{q_2^2}
\end{aligned}$$

In C.G. system



elec-elec

$3_e - 2_p$ $\quad q_1^2 = -|q_1|^2 = 2p^2(\cos\theta - 1)$

$q_2^2 = -|q_2|^2 = -2p^2(\cos\theta + 1)$

elec - pos.

$q_2^2 = -2p^2(\cos\theta + 1)$

$q_1 = 2E$

A little work yields:

elec-elec where $\gamma = \frac{E}{\mu}$

$$\sigma(\theta)\, d(\cos\theta) = \underset{\overset{\shortparallel}{K}}{\frac{\pi}{8}\frac{e^4}{m^2c^4\gamma^2}}$$

$$\text{I} \quad \times \Big\{ \frac{1}{(\gamma^2-1)^2\sin^4\theta/_2}\Big[1 + 4(\gamma^2-1)\cos^2\tfrac{\theta}{2} + 4(\gamma^2-1)^2\cos^2\tfrac{\theta}{2} + 2(\gamma^2-1)^2\sin^4\theta/_2\Big]$$

$$\text{II} \quad + \frac{1}{(\gamma^2-1)^2\cos^4\theta/_2}\Big[1 + 4(\gamma^2-1)\sin^2\tfrac{\theta}{2} + 4(\gamma^2-1)^2\sin^2\tfrac{\theta}{2} + 2(\gamma^2-1)^2\cos^4\theta/_2\Big]$$

$$\text{III} \quad - \frac{1}{(\gamma^2-1)^2\sin^2\theta}\Big[4 - 16(\gamma^2-1)^2\Big]\Big\}\, d(\cos\theta)$$

elec-pos

$$\sigma(\theta)\, d(\cos\theta) = K$$

$$\text{I} \quad \times \Big\{ \frac{1}{(\gamma^2-1)^2\sin^4\theta/_2}\Big[1 + 4(\gamma^2-1)\cos^2\tfrac{\theta}{2} + 4(\gamma^2-1)^2\cos^2\theta/_2 + 2(\gamma^2-1)^2\sin^4\theta/_2$$

$$\text{II} \quad + \frac{1}{\gamma^4}\Big[3 + 4(\gamma^2-1)(1+\sin^2\theta)\Big]$$

$$\text{III} \quad - \frac{1}{(\gamma^2-1)\gamma^2\sin^2\frac{\theta}{2}}\Big[3 + 4(\gamma^2-1)(1+\cos\theta) + (\gamma^2-1)^2(1+\cos\theta)^2\Big]\Big\}\, d(\cos\theta)$$

For $E = 300\,\text{Mev}$ $\quad \gamma^2 \sim 300$

$$K = 3.4 \times 10^{-31}\ \text{cm}^2$$

The interference term, III, is greatest at 90° in C.G. system. The following table is for this angle

(γ^2-1)	$\frac{III}{I+II}$
0	$-1/2$
$1/4$	$-12/53$
$1/2$	0
1	$3/11$
>10	$4/5$

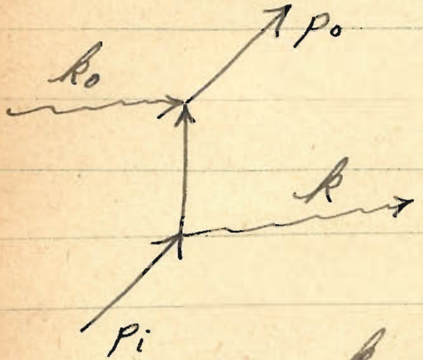
The ratio of elec-elec to elec-pos. scattering is:

$$\frac{e^- - e^-}{e^- - e^+} = \frac{1}{4} \text{ at high energy}$$
$$= 1 \text{ at low energy}$$

At 300 Mev $\lambda_{el} = \frac{1}{25} \frac{\hbar}{\mu c}$

$r_0 = \frac{1}{137} \frac{\hbar}{\mu c}$

Compton Effect   Beard

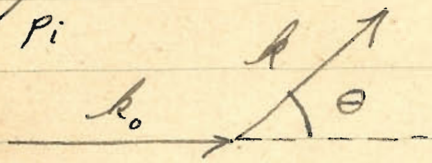


$\not{p}_o = \not{p}_i - \not{k} + \not{k}_o$

Squaring yields

$p_i \cdot k_o - p_i \cdot k = k \cdot k_o$

$= k k_o (1 - \cos\theta)$



For an electron initially at rest: $p_i = m$

$$m k_o - m k = k k_o (1 - \cos\theta)$$

$$k = m k_o / (m + k_o(1 - \cos\theta))$$

or $$\lambda - \lambda_o = \frac{1}{m}(1 - \cos\theta)$$

$$d\phi = \frac{1}{c} \cdot \frac{2\pi}{k} |H_F|^2 \rho_F$$

$$H_F = \not{e}_o \frac{1}{\not{p}_i - \not{k} - m} \not{e} + \not{e} \frac{1}{\not{p}_i + \not{k}_o - m} \not{e}_o$$

Sum over Spin yields:

$$\frac{1}{4m^2} Sp\left[H_F(\not{p}_i + m)\tilde{H}_F(\not{p}_o + m)\right]$$

Evaluation of this spur will give

the result for the particular polarizations e and e_0 of scattered and incident beam. This is a very messy calculation. It is somewhat easier to sum over all polarizations before computing the spur. Since experiments are done with unpolarized beams this is the result we want anyway.

Sum over polarizations yields

$$Sp\left\{ \left[2\gamma_\mu (\not{p}_i - \not{k} + m)\gamma_\nu(\not{p}_i + m)\gamma_\nu(\not{p}_i - \not{k} + m)\gamma_\mu \times \not{p}_0 + m \right] / 4(p_i \cdot k) + \ldots \right\}$$

<u>Useful</u> <u>relations</u> <u>for</u> <u>computing</u> <u>spurs</u>:

$$\gamma_\mu \gamma_\mu = 4$$
$$\gamma_\mu \not{A} \gamma_\mu = -2\not{A}$$
$$\gamma_\mu \not{A}\not{B}\gamma_\mu = 2(\not{A}\not{B} + \not{B}\not{A}) = 4A\cdot B$$
$$\gamma_\mu \not{A}\not{B}\not{C}\gamma_\mu = -2\not{C}\not{B}\not{A}$$

$$Sp\,(\not{A}+a)(\not{B}-b)(\not{C}+c)(\not{D}-d)$$

$$= (A\cdot B-ab)(C\cdot D-cd)+(D\cdot A-da)(B\cdot C-bc) - (A\cdot C-ac)(B\cdot D-bd)$$

Using these relations the spur we wish to compute turns out to be:

1st term $\frac{1}{2m^2}\left[\frac{p_i\cdot k_0}{p_i\cdot k}-\frac{m^2}{p_i\cdot k}+\frac{m^4}{(p_i\cdot k)^2}\right]$

2nd term $\frac{1}{2m^2}\left[\frac{p_i\cdot k}{p_i\cdot k_0}+\frac{m^2}{p_i\cdot k_0}+\frac{m^4}{(p_i\cdot k_0)^2}\right]$

The two interference terms are equal and yield

$$-\frac{1}{2m^2\,p_i\cdot k\,p_i\cdot k_0}\left(2m^4+k\cdot k_0\right)$$

Combining and inducing the energy momentum conditions yields:

$$d\phi = \frac{r_0^2}{2}\, d\Omega \,\frac{k^2}{k_0^2}\left(\frac{k}{k_0} + \frac{k_0}{k} - \sin^2\theta\right)$$

x-sec. for particular polarizations are given in Heitler p 154

<u>Bremsstrahlung</u> L. Brown

References:

A. Weizäcker - Williams Method
Heitler - p 263 2^{nd} Ed.
Weizäcker - Z.f.P. <u>88</u> 1934
Williams - Kgl. Dansk. Vid Selsk
<u>13</u> 1935

B. Bremsstrahlung
Heitler
Rossi - Greisen

C. Angular Distribution of Brem.

Hough. PR $\underline{74}$ 80

Approx valid when $E, E_0 \gg \mu$

$\theta \gg \mu/E_0$

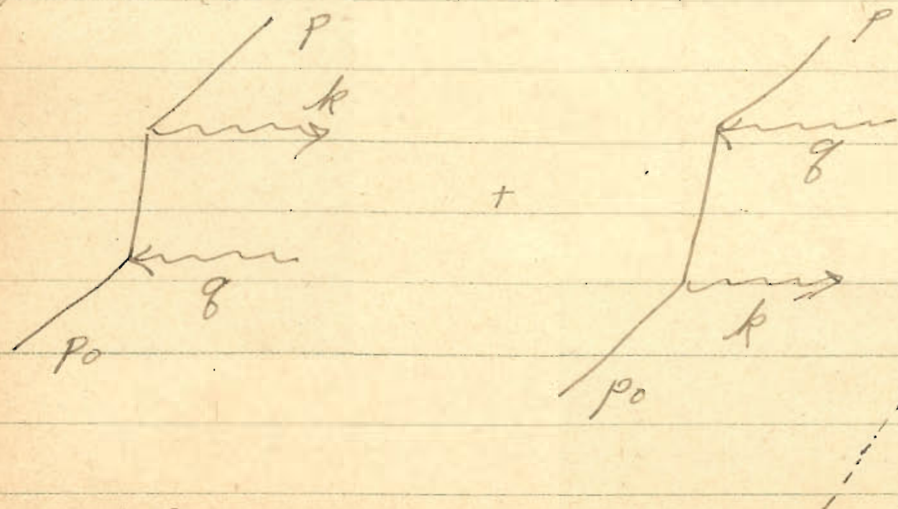
$\theta \gg \left[\left(\frac{2\mu E}{k E_0}\right) \frac{Z^{1/3}}{137} \right]^{1/2}$

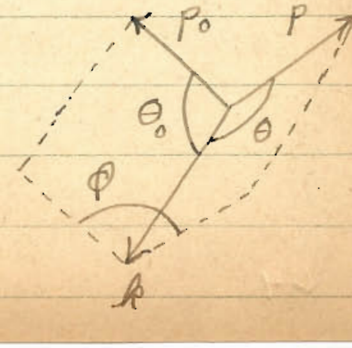
D. Brem of Mesons

Pauli RMP $\underline{13}$ 1941 p 203

Powell PR $\underline{75}$ 1949 p 32

The calculation of Bremstrahlung may be made exactly using for example the method of Feynman.

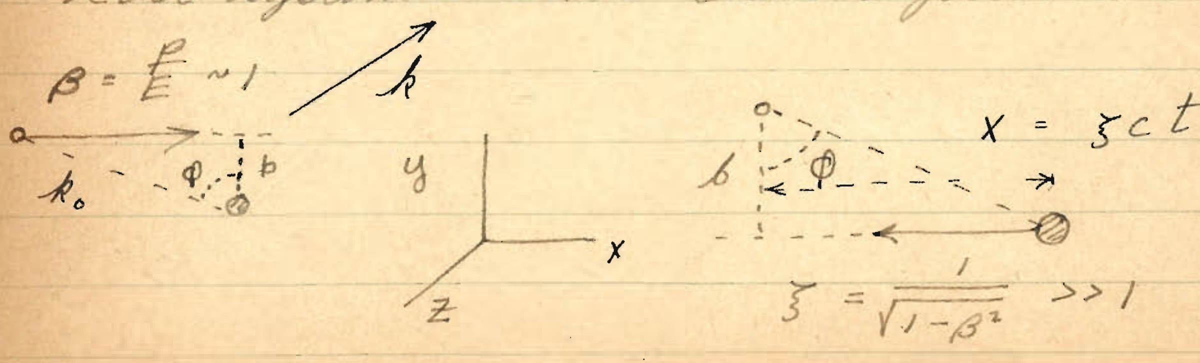




$d\Phi = d\Phi(\theta, \theta_0, \Phi, p_0, p, k)$

We shall use here the approximate method of treatment due to W & W. The method consist essentially of transforming to a coordinate system in which the electron is at rest and the nucleus is going by. The field of the nucleus can be expanded as a photon wave packet and the scattering of these photons by the electron can by calc. using the Klein Nishina formula. It is these scattered photons which in the Lab. system constitute the Bremstrahlung.

Lab system → Calc system



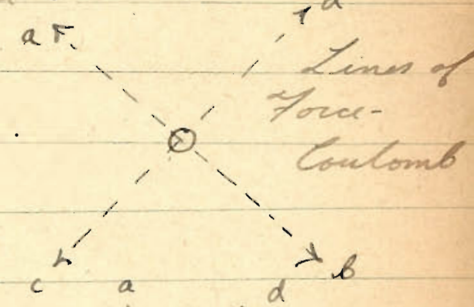
Contraction of coulomb field:

Lab

$$E_x = \frac{Ze^2}{r^2}\sin\phi$$

$$E_y = \text{"}\ \cos\phi$$

$$E_z = H = 0$$

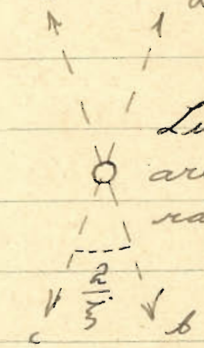


Moving

$$E'_x = \frac{Ze^2}{r^2}\sin\phi$$

$$E'_y = \xi\,\frac{Ze^2}{r^2}\cos\phi$$

$$H'_z = -\xi\beta\,\frac{Ze^2}{r^2}\cos\phi.$$



Assuming the electron recieves a small recoil, we may use the non-rel. approx. for the scattering x-section

$$\phi = \frac{8\pi}{3} r_0^{\,2}$$

valid for

$$k_0 \approx k \ll \mu$$

If $q(k_0) \equiv$ distribution of quanta in wave packet representing field, then

$$\text{x-section} = \frac{8\pi}{3} r_0^2\, q(k_0)\, dk_0$$

$$\approx \frac{8\pi}{3} r_0^2\, q(k)\, dk$$

In order to find $q(k)$, we make a Fourier analysis of the field:

$$S_x = \frac{c}{4\pi}(E \times H) = \frac{c}{4\pi} E_y'^2$$

$$\int_{-\infty}^{\infty} S_x\, dt = \frac{c}{4\pi}\int_{-\infty}^{\infty} E_y'^2\, dt$$

$$E_y'(t) = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} f(\omega_0)\cos\omega_0 t\, d\omega_0$$

$$f(\omega_0) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} E_y'(t)\cos\omega_0 t\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\, Ze^2 b\xi \int \frac{\cos\omega_0 t\, dt}{(b^2+\xi^2c^2t^2)^{3/2}}$$

= Bassett's integral: see Watson's Bessel Functions.

$$\int S_x\,dt = \frac{c}{4\pi}\int_0^\infty f^2(\omega_0)\,d\omega_0$$

Hence $\frac{c}{4\pi} f^2(\omega_0)$ = energy distribution at b

and

$$\frac{1}{\omega_0}\frac{c}{4\pi} f^2(\omega_0)\,d\omega_0 = \#\text{ photons at } b.$$
$$= p(k)\,dk$$

Therefore:

$$q(k)\,dk = dk\int_{b_{min}}^\infty p(k)\,2\pi b\,db$$

= # of equivalent quanta in field.

In this calc, it is assumed that the relative sideways velocity between the two particles is $\ll c$. This implies $p_b < mc$, and from the uncertainty principle one obtains:

$$b\,p_b \geq \hbar$$
$$b \geq \hbar/mc = b_{min}$$

here m is the mass of the lighter particle. The integration is extended only down to b_{min}. The contribution from smaller b is negligible.

Ref. Heitler & Peng - Proc. Roy. Irish Acad.

One now integrates over all equivalent quanta with k greater than that for which one wishes to compute the Brem. and transforms back to the lab. system.

One obtains:

$$\Phi(k)\,dk = \frac{4\pi r_0^2}{137} Z^2 \frac{E'}{E}\left[\left(\frac{E}{E'} + \frac{E'}{E} - \frac{2}{3}\right)\right.$$

$$\left.\times\left(\ln\frac{2EE'}{\mu k} - 1.4\right) - \frac{1}{9}\right] \quad : \; k \gg \mu$$

$E' = E - k$ $\qquad$ E = energy of electron

The above was calc using for the K.N. formula

$$d\Phi = \pi r_0^2 \mu \frac{dk}{k_0^2}\left(\frac{k}{k_0} + \frac{k_0}{k}\right)$$

The approx formula here given differs from the exact in that $\frac{1}{9} \to 0$

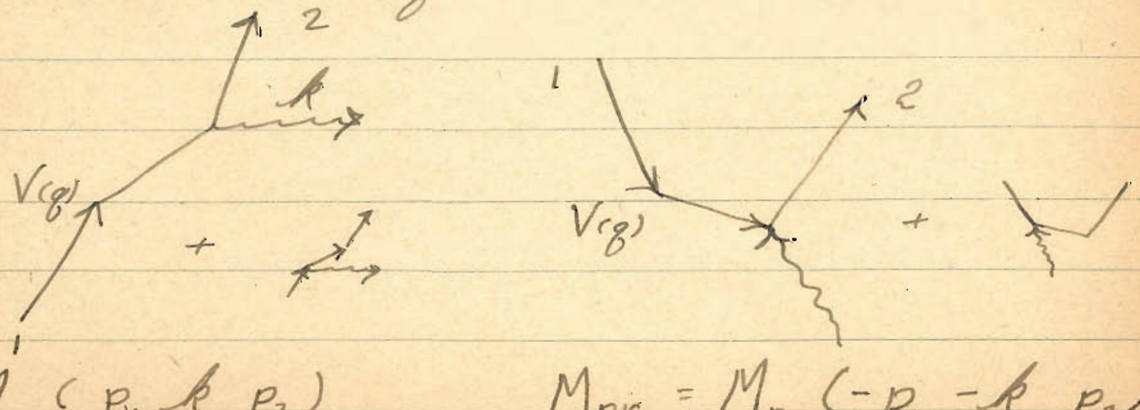
$1.4 \to .5$

For Brem of mesons see Pauli

Application of theory to bursts
Christi & Kusaka PR $\frac{59}{60}$? 1941

<u>Pair</u> <u>Production</u> : Trammel

Using Feynman approach pair production in the field of a nucleus is equivalent to Brem.



$M_{Br}(p_1\ k\ p_2)$ $\qquad$ $M_{pr} = M_{Br}(-p_1\ -k\ p_2)$

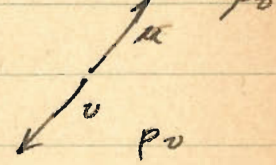
Ref: Heitler
Hough PR 74 80

<u>Showers</u> - Schwartz

Ref. Rossi - Greisen.

Feynman

$\pi \to \mu + \nu$



Coupling $G_{\pi\mu} \cdot 1$

Prob. of disint / sec $= \frac{1}{\tau_{\pi\mu}}$

$$= \frac{2\pi}{\hbar} \frac{G_{\pi\mu}^2}{m_\pi} \frac{4\pi p_\nu^2 \, dp_\nu}{(2\pi\hbar)^3 \, dE}$$

$$E = E_\nu + E_\mu$$
$$= \sqrt{m_\mu^2 + p_\nu^2} + p_\nu$$
$$\frac{dE}{dp} = p_\nu / \sqrt{m_\mu^2 + p_\nu^2} + 1 \simeq 1$$
$$p_0 = m_\pi - \sqrt{m_\mu^2 + p_\nu^2} \simeq m_\pi - m_\mu$$
$$\frac{1}{\tau_{\pi\mu}} = \ldots \frac{1}{\pi} \frac{G_{\pi\mu}^2}{\hbar c} \left(\frac{m_\pi c^2}{\hbar} \right) \left(\frac{m_\pi - m_\mu}{m_\pi} \right)^2$$

Capture of μ meson

$$\mu \to \pi + \nu$$
$$\pi + \text{nucleus} \to \text{nucleus}^*$$

Use old perturbation theory:

Inter. state $\equiv \pi + \nu$

Neutrino energy $= \epsilon_\nu = \mu - E$

E = diff in energy of initial and final nuclei

Inter. state energy denom.

$$m_\pi + \epsilon_\nu - \mu = m_\pi - E$$

$$M_{fi} = g\left[\int \Psi_f\, \tau_-\, \Psi_i\, dV\right] \frac{1}{m_\pi - E}\, \frac{G_{\pi\mu}}{m_\pi}\, \varphi(0)$$

This may be read back to front as: amplitude for μ at nucleus $\times$ amplitude $\mu \to \pi + \nu$ $\times$ life time of virtual $\pi + \nu$ $\times$ amplitude for absorption of π by nucleus.

$$\text{Prob / sec} = \frac{2\pi}{\hbar}\, \frac{g^2}{\hbar c}\left(\frac{G_{\pi\mu}^2}{\hbar c}\right) \frac{4\pi \epsilon_\nu^2\, d\epsilon_\nu}{m_\pi^2 (m_\pi - E)^2}\, \frac{\rho_E\, dE}{d\epsilon}\, \frac{1}{(2\pi\hbar)^3}$$

$\rho_E\, dE$ = density of energy levels in final nucleus $\times\, dE$

Prob / sec for capture with energy diff of initial and final nuclei $E - E + dE$

$$= \ldots \frac{1}{\pi}\frac{g^2}{\hbar c}\left(\frac{G_{\pi\mu}}{\hbar c}\right)^2 \frac{1}{m_\pi^2}\left(\frac{m_\mu - E}{m_\pi - E}\right)^2 \rho_E M_E \, dE$$

Total rate $= \int (\text{Prob/sec})\, d\epsilon_v$

$$= \ldots \int \left(\frac{m_\mu - E}{m_\pi - E}\right)^2 \rho_E M_E \, dE$$

$$\approx \ldots \left(\frac{m_\mu}{m_\pi}\right)^2 \int \left(1 - \frac{E}{Mev}\right)^2 \rho_E M_E \, dE$$

$$= Z(\psi_i^* \psi_i)$$

$$\int \rho_E M_E \, dE = Z \sum (\psi_i^* \tau_+ \psi_f)(\psi_f^* \tau_- \psi_i)$$

$$\int \rho_E M_E E \, dE = Z \sum (\psi_i^* \tau_+ \psi_f)(\psi_f^* \tau_- \psi_i) \Delta E$$

$$= Z \{(\psi_i^* \tau_+ H \tau_- \psi_i) - (\psi_i^* H \psi_i)\}$$

$$= Z E_{av}$$

E_{av} = diff in energy if one proton turned into a neutron in place $\sim$ 10 Mev.

$$\approx \ldots Z\left(\frac{m_\mu}{m_\pi}\right)^2 \left(1 - \underbrace{\frac{E_{av}}{Mev}}_{\text{small}}\right)^2$$

$$\frac{1}{\tau_{cap.\mu}} = \frac{1}{\tau_{\pi\mu}} \left(\frac{g^2}{\hbar c}\right) \frac{4 m_\mu^5 Z (Z/137)^3}{m_\pi^3 (m_\pi - m_\mu)^2}$$

Data from Berkley

$$m_\pi = 284 \pm 6 \; m_e$$

$$\frac{m_\pi}{m_\mu} = 1.31 \pm .01$$

$$\tau_{\pi \to \mu} = .7 \times 10^{-8} \text{ sec} \quad -$$
$$.6 \times 10^{-8} \text{ ''} \quad +$$

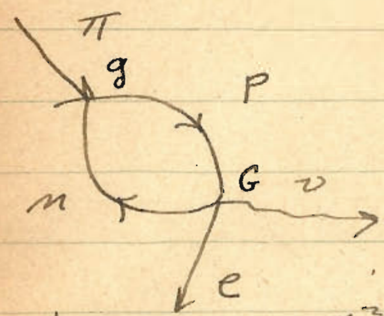
$$\tau_{\mu \to e} = 2.19 \times 10^{-6} \text{ sec}$$

In μ capture, the ν takes away most of the energy. A neutron comes off and in the region where experiments have been done this leaves a stable nucleus.

β-decay

Some possibilities

I



$$g = G = 1$$

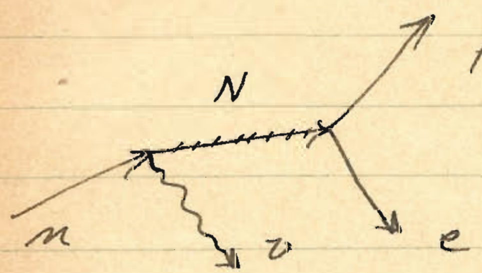
$$I = \iint \mathrm{Sp}\, \frac{1}{\not p + \not \pi - M}\, 1\, \frac{1}{\not p - M}\, 1 \Big\} dp$$

div. quad., cut off.

$$\frac{1}{\tau_{\pi \to e + \nu}} = \frac{g^2}{\hbar c} \left(\frac{1}{\tau_{N \to e+\nu}} \left(\frac{mc^2}{W} \right)^5 \right) \left(\frac{I}{m_\pi^2} \right)^2$$

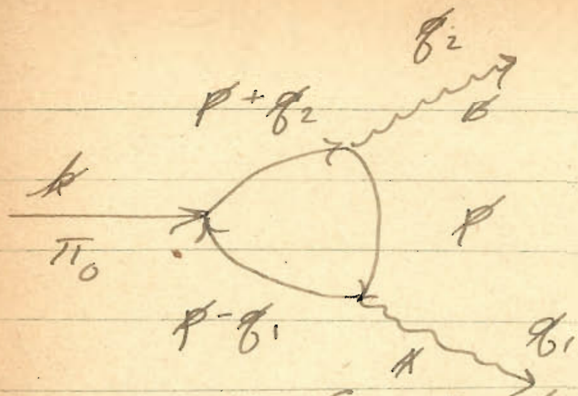
$$W = 750\,\mathrm{Kv.}$$

II Process involving new particle



Neutral Mesons

If one bombards Be with D, one observes more γ's at high bombarding energies than can be accounted for by Bremsstrahlung. One possible explanation is that π_0 are produced which decay $\pi_0 \to 2\gamma$



$k = q_1 + q_2$

$$M = g \int Sp\left[\frac{1}{\not{p} - \not{q} - M} \not{A} \frac{1}{\not{p} - M} \not{B} \frac{1}{\not{p} + \not{q}_2 - M} \Gamma\right] d^4p$$

which may be reduced to

$$M = \dots g \int Sp\left[\not{q}_1 \not{A} \not{B} \not{q}_2 \Gamma \right] d^4p$$

Life time turns out $\sim 10^{-19}$ sec.

Lateral Spread of Showers R Thompson

Ref. Eyges P.R. Dec 15 1948