Technique for Measuring the Electrical Conductivity of Wakes of Projectiles at Hypersonic Speeds*

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A technique is described for measuring the electrical conductivity of hypersonic wakes and any conducting medium by measurement of the Joule losses produced by the oscillating magnetic field of a circular coil surrounding it. The apparatus consists of a symmetrical rf bridge, two arms of which contain identical coils. A conducting medium inserted into one of the coils unbalances the bridge, changing the apparent impedance of the coil. Although the apparatus was designed specifically to investigate the conductivity of hypersonic wakes, it has also been used to measure the conductivity of electrolytic solutions, electrical discharges, flames, and plasmas produced in shock tubes. The measurements on electrolytic solutions gave results in satisfactory agreement with their known conductivities and provided a convenient check on the calibration of the apparatus. The shock tube measurements were made in air and agreed well with the results of Lamb and Lin [L. Lamb and S. C. Lin, J. Appl. Phys. **28**, 754 (1957)]. Results of the measurements of the conductivity of electrolytic solutions, shock tube heated plasmas, and hypersonic wakes of 0.22-in.-diam nylon spheres in argon are presented.

INTRODUCTION

THIS paper describes a simple technique for determining the electrical conductivity of a medium from measurements of the Joule losses produced by the oscillating magnetic field of a circular coil surrounding it. The technique is similar in principle to that developed by Lin, Resler, and Kantrowitz[1] to measure the conductivity behind shock waves in argon, but has the advantage that it may be employed in cases where the medium is stationary. Although the apparatus was designed specifically to investigate the conductivity in the wake of hypersonic pellets, it has also been used to measure the conductivity of electrolytic solutions, electrical discharges, flames, and plasmas produced in shock tubes. The measurements on electrolytic solutions gave results in satisfactory agreement with their known conductivities and provided a convenient check on the calibration of the apparatus. The shock tube measurements were made in air and agreed well with the results of Lamb and Lin.[2]

THEORY OF OPERATION

A schematic diagram of the apparatus designed to measure wake conductivities is shown in Fig. 1. It consists of a symmetrical rf bridge, two arms of which contain identical coils. When a conducting medium is inserted into one of the coils, currents are induced due to the oscillating magnetic field. As a result, the apparent impedance of the coil changes and the bridge becomes unbalanced. In general, one must measure both the amplitude and phase of the bridge output voltage to determine the electrical properties of the medium. However, in the present application both the displacement current and the skin effect in the medium were negligible, so that the change in the impedance of the coil was entirely resistive and the conductivity σ could be determined by measuring only the amplitude of the output voltage. For the displacement current to be negligible,

$$\sigma \gg \omega\epsilon, \tag{1}$$

where ω is the angular frequency of the magnetic field and

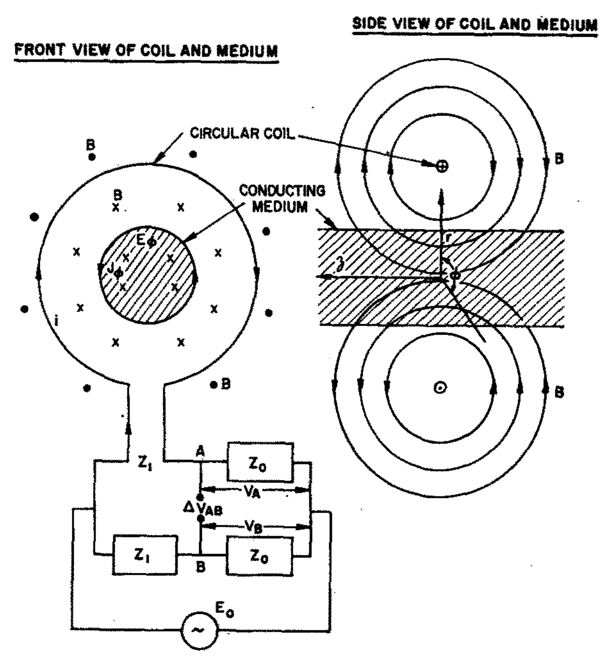

FIG. 1. Principle of electrical conductivity wake measurement. Shown are the geometry and coordinate system used in computing the resistive change of the coil due to any cylindrically symmetrical conducting medium and the bridge circuit used to measure this change. **B** is the oscillating magnetic field which produces the electric field E_ϕ and the current J_ϕ in the wake. The Z's are the bridge impedances and E_0 is the oscillator voltage. ΔV_{AB} is the bridge unbalance due to the wake or plasma.

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[1] S. C. Lin, E. L. Resler, and A. Kantrowitz, J. Appl. Phys. **26**, 95 (1955).
[2] L. Lamb and S. C. Lin, J. Appl. Phys. **28**, 754 (1957).

ϵ is the dielectric constant of the medium; for the skin effect to be negligible,

$$\sigma \ll 2(\mu\omega r_w^2)^{-1}, \qquad (2)$$

where μ and r_w are the magnetic permeability and characteristic radius of the medium. Under the above conditions the change in the impedance of the coil is

$$\Delta R = P/I^2, \qquad (3)$$

where I is the rms current in the coil and P is the power dissipated in the medium.

If we assume that the conductivity has cylindrical symmetry about the axis of the coil,

$$P = \int_0^{+\infty}\int_{-\infty}^{+\infty} \sigma E_\phi^2 2\pi r\, dr dz, \qquad (4)$$

where

$$E_\phi = \partial A_\phi/\partial t = \omega A_\phi \qquad (5)$$

is the electric field,

$$A_\phi = (\mu NI/2\pi)(r_c/r)^{\frac{1}{2}} k^3 C(k^2) \qquad (6)$$

is the vector potential for a circular coil of N turns, $k^2 = 4rr_c/[(r_c+r)^2+z^2]$, r_c is the radius of the coil and $C(k^2)$ is a complete elliptic integral[3] which for small values of k^2 may be approximated

$$C(k^2) = \tfrac{1}{16}\pi(1+\tfrac{3}{4}k^2+\cdots). \qquad (7)$$

In writing Eq. (5) we have tacitly assumed that the axial velocity of the medium

$$v_z \ll r_c\omega \qquad (8)$$

so that the electric field arising from the radial component of the magnetic field may be neglected. We have also assumed that the electrostatic field of the coil has been terminated by an appropriately designed Faraday shield.

Combining Eqs. (3)–(6) we obtain the expression

$$\Delta R = \frac{3}{4r_c}\left[\frac{\pi\mu\omega N}{4}\right]^2 \int_0^{+\infty} \bar{\sigma}(r) r^3 dr, \qquad (9)$$

where

$$\bar{\sigma}(r) = \frac{8}{3\pi}\left(\frac{16}{\pi}\right)^2 \int_{-\infty}^{+\infty} \frac{\sigma(r,z)C^2(k^2)r_c^5 dz}{[(r_c+r)^2+z^2]^3} \qquad (10)$$

is a weighted average of the conductivity with respect to z. In many cases of practical importance σ will be large for $r \ll r_c$ and $\bar{\sigma}(r)$ may be approximated, using Eq. (7), as

$$\bar{\sigma}(r) \approx \frac{8}{3\pi}\int_{-\infty}^{+\infty} \frac{\sigma(r,z)r_c^5 dz}{(r_c^2+z^2)^3}. \qquad (11)$$

[3] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, Inc., New York, 1945), p. 73.

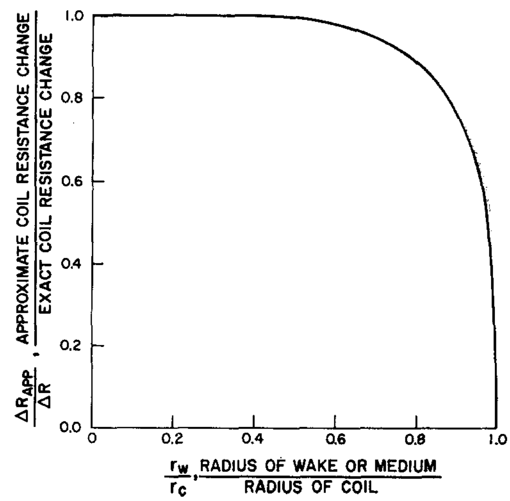

FIG. 2. Comparison of the coil resistance change using the approximate expression for $r \ll r_c$ with that using the exact expression. r_c is the coil radius.

The accuracy of this approximation may be judged from Fig. 2 which shows that it is good to within 25% out to a radius of $r/r_c = 0.90$. Note that the effective axial resolution for the system is approximately the coil diameter $2r_c$, and is determined by the function

$$f = (8/3\pi)(r_c^5/[r_c^2+z^2]^3), \qquad (12)$$

where

$$\int_{-\infty}^{+\infty} f dz = 1.$$

Under conditions where the axial variation of conductivity is small over a distance equal to r_c, $\bar{\sigma}(r) = \sigma(r,0)$.

An alternative form for ΔR, which may be obtained by integrating Eq. (9) by parts, is

$$\Delta R = 3/r_c(\pi\mu\omega N/16)^2 \bar{\sigma}(0)\bar{r}_w^4, \qquad (13)$$

where

$$\bar{r}_w = \left[\frac{1}{\bar{\sigma}(0)}\int_0^{+\infty} r^4 \frac{d\bar{\sigma}(r)}{dr} dr\right]^{\frac{1}{4}} \qquad (14)$$

is the effective radius of the conducting medium threading the coil. For a constant conductivity medium of radius a, $\bar{\sigma}(r) = \bar{\sigma}(0)$ for $r < a$ and $\bar{\sigma}(r) = 0$ for $r > a$ and $\bar{r}_w = a$ from Eq. (14), while for a Gaussian conductivity, $\sigma = \sigma(0)e^{-r^2/a^2}$, the effective radius is $\bar{r}_w = (2)^{\frac{1}{2}}a$. Equation (13) is the basic relation used in interpreting the measurements.

EXPERIMENTAL APPARATUS

Bridge Circuit

A photograph of the apparatus used for measuring conductivity and a schematic diagram of the bridge circuit are shown in Fig. 3. The bridge was designed to be symmetrical and consists of two arms, each having a circular coil in series with a resistor. The coils are connected to the bridge by two coaxial cables as seen in the photograph in Fig. 3. This allows for separation of the coils from each other and the bridge box, which contains the balancing elements and other components. The balancing elements are a capacitor C_v for balancing the coils, as well as any capacitive differences between the arms, and a potentiometer R_p for balancing the resistive differences in the arms. The output of the bridge ΔV_{AB} is coupled through a transformer into an amplifier and then to an oscilloscope through a filter. The amplification of the transformer-amplifier-filter-'scope treated as a unit is 14.6, and the oscillator is crystal controlled at 3.800 Mc.

Under the condition that

$$\Delta R \ll |Z_0 + Z_1|, \tag{15}$$

the bridge output is given by the expression

$$\Delta V_{AB} = (|Z_0| / |Z_1 + Z_0|^2) E_0 \Delta R. \tag{16}$$

For the circuit shown, $|Z_0| = 147\ \Omega$ and $|Z_1 + Z_0| = 60\ \Omega$. Since in practice ΔR was usually considerably less than $0.01\ \Omega$, Eq. (15) was easily satisfied, and Eq. (16) is an excellent approximation. Combining Eqs. (13) and (16),

$$\Delta V_{AB} = \left(\frac{|Z_0|}{|Z_1 + Z_0|^2} E_0 \right) \left(\frac{3}{r_c} \left(\frac{\pi \mu \omega N}{16} \right)^2 \right) \bar{\sigma}(0) \bar{r}_w^4, \tag{17}$$

and at the 'scope $\Delta V = 14.6 \Delta V_{AB}$.

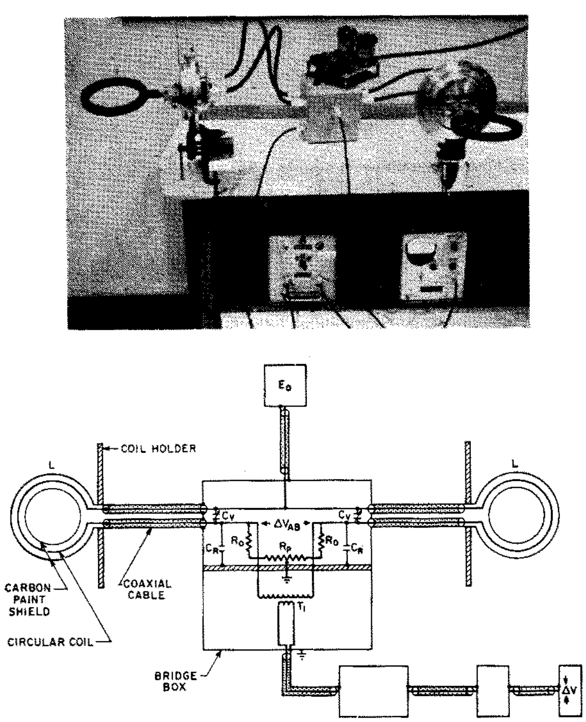

FIG. 3. Circuit diagram and photograph of conductivity bridge used to measure wake conductivity. Shown are the oscillator, amplifier, coils, and bridge box.

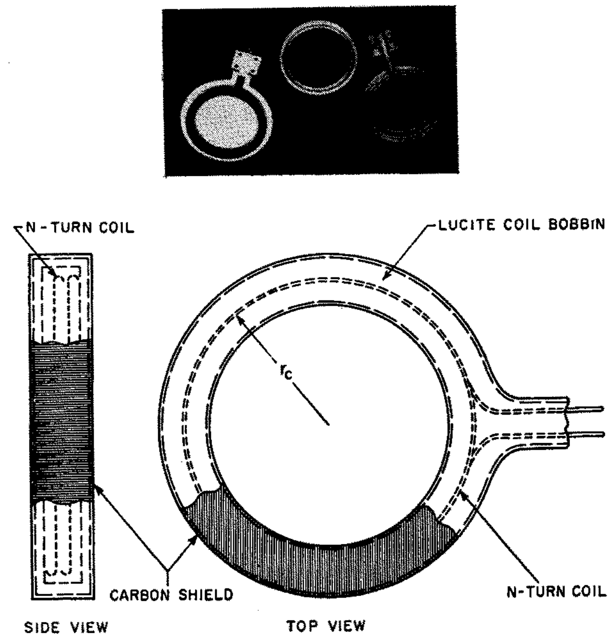

FIG. 4. Sketch and photograph of circular coil used in conductivity bridge to measure wake conductivity.

An important property of the bridge design is its symmetry. This allows one to determine whether an unbalance of the bridge produced by the conducting medium is capacitive, inductive, or resistive by causing the particular type of unbalance in the passive coil and observing whether the effect of the conducting medium in the active coil tends to balance or further unbalance the bridge.

Coil Design

A schematic diagram of the coil designed to measure conductivity and a photograph of its components are shown in Fig. 4. The coil was constructed of 4 turns of wire wound on a circular Lucite bobbin, 5.5 in. in diameter and inserted into a Lucite "lollipop" support. The Faraday shield used to eliminate pickup and terminate the electrostatic field of the coil was built up of 30 coats of carbon paint (very low resistance carbon paint, RO1, manufactured by Microcircuits Company, New Buffalo, Michigan) applied over the entire "lollipop" and protected first by a "Q" dope covering and then by an epoxy covering.

The resistance and inductance of this coil were, respectively, $2.3\ \Omega$ and $6.5\ \mu$H. The Q of the coil, including losses and the Faraday shield, was 68.12 at the operating frequency of 3.8 Mc, and the coil had its natural resonance at 13.0 Mc.

Two considerations were involved in the design of the Faraday shield. First, to allow the magnetic field of the coil to penetrate the shield,

$$d_s \ll \delta_c = 2(\sigma_c \mu \omega)^{-\frac{1}{2}}, \tag{18}$$

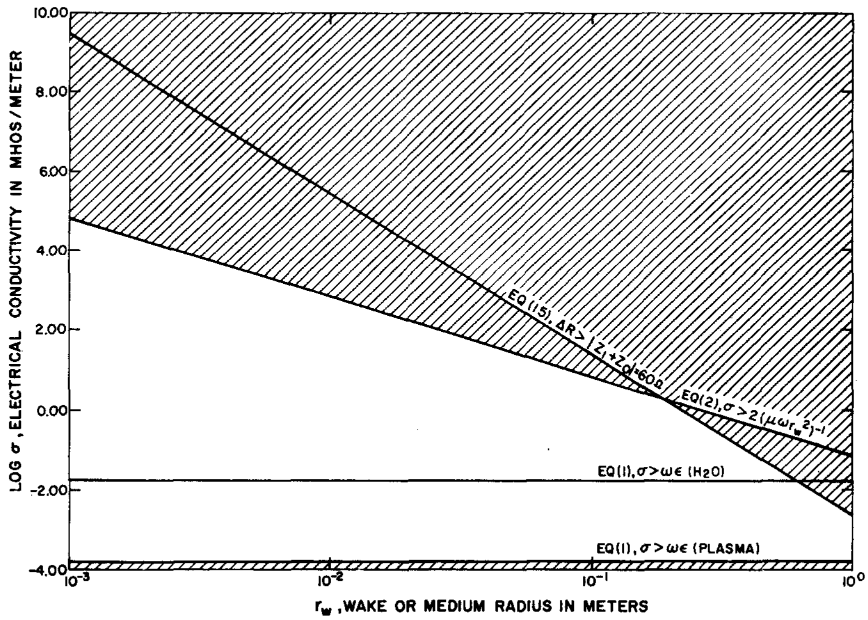

FIG. 5. Conductivity limits for conductivity bridge. The cross hatched area is the region where the skin and displacement current effects are comparable to the conduction current effect, and the change of the coil resistance is comparable to the bridge arm impedance.

where d_s is the thickness of the shield and δ_c is the skin depth in the carbon paint. Second, for the shield to be effective, its relaxation time

$$RC \ll (\omega)^{-1}, \tag{19}$$

where R is the resistance ($1.83\times10^{+1}\Omega$) of the shield and C is the capacity ($60\ \mu\mu$F) of the shield with respect to the coil. For coils of the type shown in Fig. 4, Eqs. (18) and (19) required $d_s \ll 1.16$ in. and $RC \ll 4.19\times10^{-8}$ sec, respectively. Since the actual thickness of the shield was $d_s = 0.098$ in. and $RC = 1.10\times10^{-9}$ sec, both these criteria were easily satisfied.

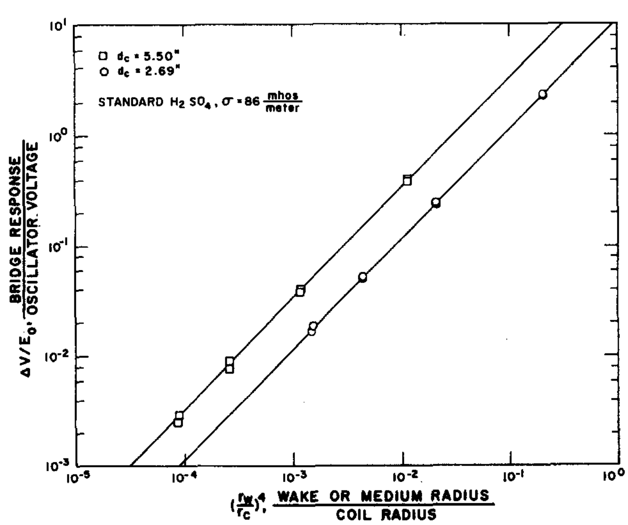

FIG. 6. Experimental verification of the coil resistance change dependency on $\bar{r}_w^4$ and the approximation for $r \ll r_c$. The linearity and slope of 1 on this log-log plot verifies the combination of Eqs. (11) and (17).

Calibration

The range of conductivities that may be measured with the bridge described above is determined by the conditions given in Eqs. (1), (2), and (15) and is shown in Fig. 5 as a function of the effective radius of the conducting medium. For conductivity in this range, a convenient method of calibrating the bridge was to use standard electrolytic solutions of H_2SO_4 in glass tubes. The results of such a calibration are given in Fig. 6 which shows a log-log plot of $\Delta V/E_0$ as a function of $(r_w/r_c)^4$, where r_w is the tube radius. The linearity and slope of 1 of the plot verifies the predictions of Eqs. (11) and (17), and the measured conductivity of 96 mhos/m at a temperature of 22°C is in satisfactory agreement with the value obtained from the *Handbook of Chemistry and Physics*[4] for the dc conductivity

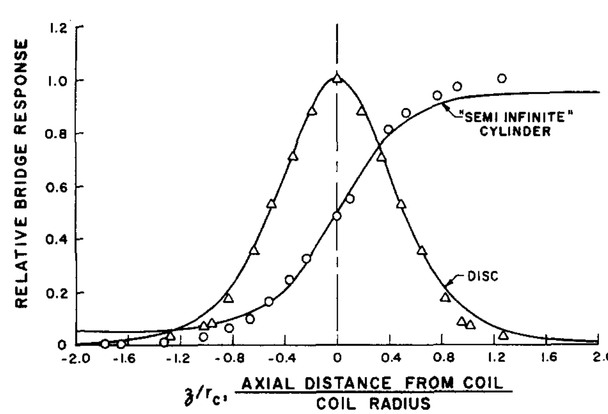

FIG. 7. Comparison of theory and experiment for axial field dependence to determine the effective beam width. Shown are the results for a "semi-infinite" tube and disk of standard H_2SO_4 solution.

[4] *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Company, Cleveland, Ohio, 1959–1960), 41st ed., p. 2606.

of the solution used. An investigation of the variation of solution conductivity with frequency[5] indicated that the difference between the dc value and the value at the bridge frequency was negligible.

The resolution function for the coil was explored using both "semi-infinite" tubes and thin disks of electrolytic solution. The results are plotted in Fig. 7 which shows the relative output of the bridge as a function of position of the thin disk or end of the "semi-infinite" cylinder. The curves in the figure were computed from Eq. (12) showing the expected resolution functions for the two cases, and the agreement with the measurements is satisfactory.

EXPERIMENTAL MEASUREMENTS

Shock Tube Results

To check the over-all performance of the conductivity bridge, it was used to measure the conductivity of air

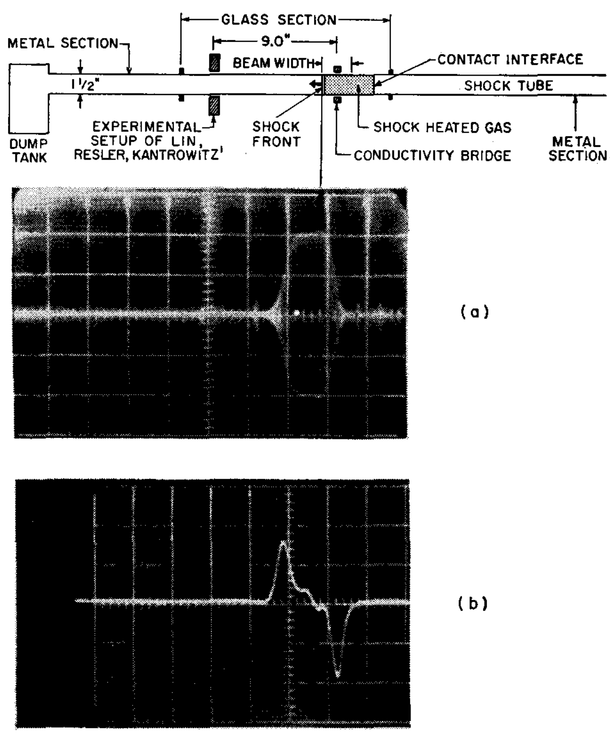

FIG. 8. Typical oscillograms obtained simultaneously in shock heated air in a $1\frac{1}{2}$-in. shock tube at an initial pressure of 1 cm Hg with the conductivity bridge with $r_c = 1.12$ in. [oscillogram (a)] and the technique of Lin, Resler, and Kantrowitz (Ref. 1) [oscillogram (b)]. Shown also is a schematic of the experimental setup. The shock is positioned in the schematic to correspond to the signal it produces on the oscillogram. Note the conductivity bridge measures σ while the technique of Lin, Resler, and Kantrowitz measures $d\sigma/dz$. For oscillogram (a) the gain is 100 mV/div, the sweep speed is 20 μsec/div, the shock velocity is 5.06 mm/μsec, and the conductivity bridge sensitivity $\Delta V/\sigma r_w^4$ is $3.30\times 10^{+7}$ mV/(mhos$\cdot$m^{-1})m^4. For oscillogram (b) the gain is 50 mV/div, the sweep speed is 20 μsec/div, the shock velocity is 5.00 mm/μsec, and the sensitivity given by the current is 241 A. The difference in velocity is due to shock attenuation between the two stations.

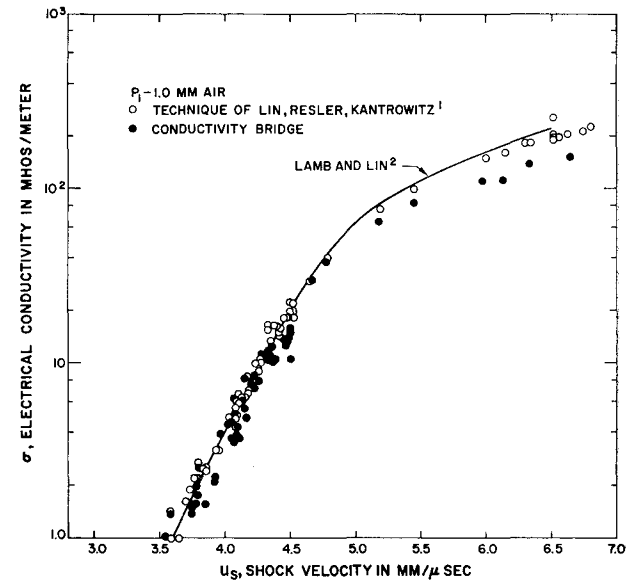

FIG. 9. Electrical conductivity of shock heated air in a $1\frac{1}{2}$-in. shock tube at an initial pressure of 1 mm Hg measured simultaneously by the conductivity bridge technique ($r_c = 1.12$ in.) and the technique of Lin, Resler, and Kantrowitz (Ref. 1) (see Fig. 8 for schematic of experimental setup). The earlier results of Lamb and Lin (Ref. 2) are also shown.

ionized by shock waves in a $1\frac{1}{2}$-in. combustion driven shock tube. A glass tube was inserted between two metal sections of the shock tube and the active coil placed around this glass section. Downstream of the conductivity bridge coil, the apparatus of Lin, Resler, and Kantrowitz[1] was placed. Figure 8 shows a sketch of the shock heated gas moving through the glass section and typical oscillograms for both techniques. The top oscillogram shows the bridge output voltage ΔV, and the bottom oscillogram shows the output of the apparatus of Lin, Resler, and Kantrowitz. Note that the amplitude of the conductivity bridge signal is proportional to the conductivity σ, while the signal from the apparatus of Lin, Resler, and Kantrowitz is proportional to $d\sigma/dz$. The finite rise times of the signals are determined by the resolution of the measuring techniques which, in both cases, is of the order of the coil diameter divided by the shock speed. The rise time is approximately 11 μsec which, at a shock speed of 5.06 mm/μsec, corresponds to 2.19 in., the diameter of the coil used.

A comparison of the results of the two techniques is shown in Fig. 9, along with the curve representing earlier results obtained by Lamb and Lin.[2] The agreement is excellent and demonstrates the ability of the conductivity bridge to make time resolved measurements with microsecond resolution.

Wake Measurement

Experiments were conducted on the Avco-Everett Research Laboratory Ballistic Range in which nylon spheres 0.22 in. in diameter were fired through argon at

[5] Samuel Glasstone, *Introduction to Electrochemistry* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1956), Chap. III.

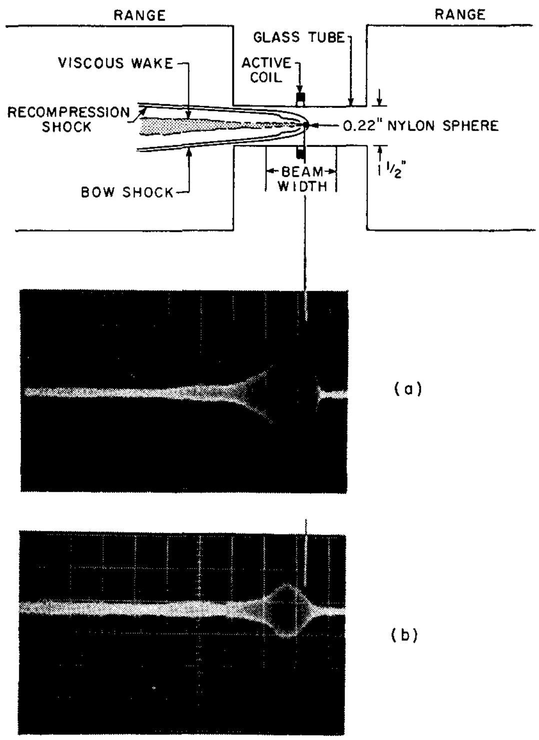

FIG. 10. Oscillograms showing response of conductivity bridge (r_c=1.38 in.) to a hypersonic 0.22-in.-diam nylon sphere in argon at 6 cm Hg pressure. Shown also is a schematic of the experimental setup. The sphere is positioned in the schematic to correspond to the signal it produces on the oscillograms. For oscillograms (a) and (b) the gain is 5 mV/div, the sweep speed is 20 μsec/div, and the conductivity bridge sensitivity, $\Delta V/\sigma r_w^4$, is $1.86\times10^{+8}$ mV/(mhos·m^{-1})m^4. The sphere velocity is 14 900 ft/sec for (a), and 12 800 ft/sec for (b).

60 mm Hg pressure at velocities between 12 000 and 15 000 ft/sec. As in the shock tube experiment, a glass tube $1\frac{1}{2}$ in. in diameter was inserted between two sections of the range, and the active coil placed around this glass section. Figure 10 shows a sketch of the projectile moving through the glass section together with two typical oscillograms obtained at velocities of 12 800 and 14 700 ft/sec. It is seen that the initial rise is the effective beam width, d_c=2.76 in. or 25 body radii, but that the decay is longer indicating that the decay is caused by a conductivity decrease. Using Eq. (17), values of $\bar{\sigma}(0)\bar{r}_w^4$ have been obtained as a function of distance behind the pellet, and the results are shown in Fig. 11 for several shots fired at velocities in the vicinity of 13 500 ft/sec.

To obtain an estimate of the wake conductivity $\bar{\sigma}(0)$ from the data of Fig. 11, the effective electronic radius $\bar{r}_w$ of the wake must be known. Although no direct measurement of this parameter is available, extensive studies of wake radii have been made using an optical technique,[6] and a summary of these results is shown in Fig. 12. If we assume that the electronic radius is the same as the optical radius, we obtain the conductivities shown in Fig. 13. We have confined ourselves to that region of the wake where (1) the axial variation of the wake properties is small, and (2) luminous wake data exist (Fig. 12). Both criteria are satisfied for $x/r_n\geq20$. Also shown is a curve of the conductivity given by Spitzer[7] for a fully ionized gas at the estimated wake temperatures. The observations lie considerably below this curve, indicating that the conductivity is in a region where it should be proportional to the degree

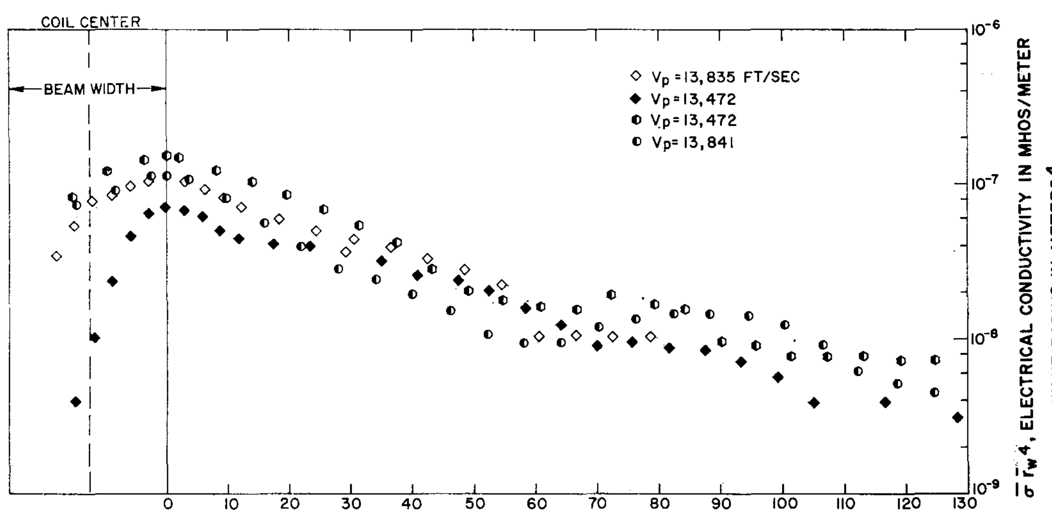

FIG. 11. Experimental results obtained with the conductivity bridge (r_c=1.38 in) for a hypersonic 0.22-in.-diam nylon sphere in argon at 6 cm Hg pressure. Shown are 4 runs at approximately 13 500 ft/sec. The amplitude of oscillograms like those of Fig. 10 has been converted into the product of the wake conductivity and the fourth power of the effective wake radius.

[6] R. L. Taylor, J. C. Keck, B. W. Melcher II, and R. M. Carbone, AIAA J., 1, 2186 (1963).
[7] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), pp. 81–86.

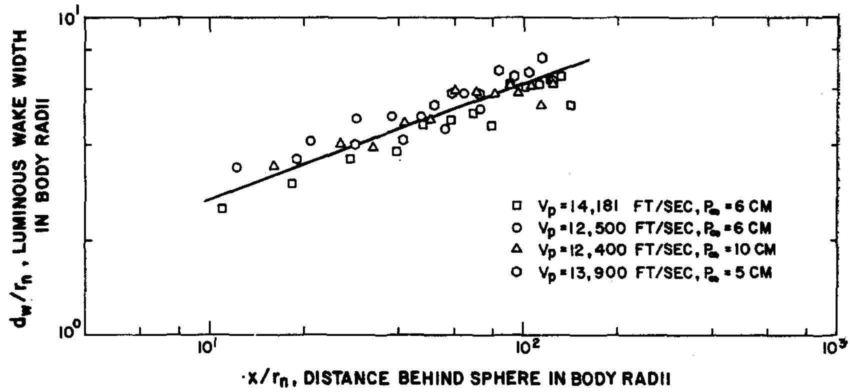

FIG. 12. Luminous wake growth for a series of runs of a 0.22-in.-diam nylon sphere in argon at argon at 5.0 cm Hg–10.0 cm Hg pressure obtained by measuring the wake self-luminescence using the wake scanner technique (Ref. 6). This technique scans the wake radially measuring the light by means of a photomultiplier.

of ionization. If the ionization were in equilibrium, we would expect the conductivity variation given by the curve labelled "equilibrium." It can be seen that the conductivity falls much more slowly than this, indicating a non-equilibrium condition in which recombination is occurring. Assuming that the recombination involves predominately binary collisions between ions and electrons in a process of the type

$$X_2^+ + e \rightarrow 2X, \tag{20}$$

and including expansion due to wake growth, we obtain the rate equation

$$dn_e/dt = -\alpha n_e^2 - n_e(d\ln A/dt), \tag{21}$$

where n_e is the electron density and A is the area of the wake πr_w^2. The equation for the luminous wake growth shown in Fig. 12 is

$$r_w/r_n = 0.575(x')^{0.368}, \tag{22}$$

where r_n is the pellet radius and $x'=x/r_n$ is the distance behind the pellet in body radii. The solution to Eq. (21) for $x'\gtrsim 20$, using Eq. (22) and substituting σ for n_e, is

$$\sigma = (2.81\times10^{-2})\left\{3.31\times10^{-1}\pi r_n^2 \nu x'^{0.736} \times\left[\frac{\alpha}{u}\,\frac{x'^{0.264}-2.21}{8.74\times10^{-2}\pi r_n}+C\right]\right\}^{-1}, \tag{23}$$

where ν is the electron-molecule collision frequency, u is the pellet velocity, and

$$C = (9.37\times10^{-3}/\pi r_n^2 \sigma_{20}\nu_{20}).$$

As can be seen in Fig. 13, Eq. (23) provides an excellent fit to the data with $\alpha = 3.5\times10^{-7}$ cm³/part-sec which is typical

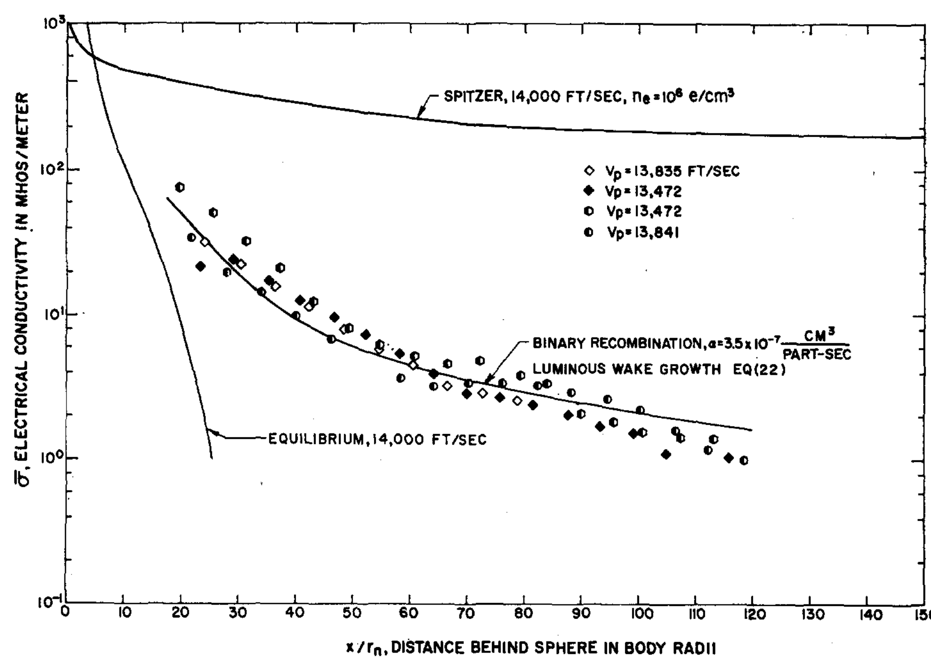

FIG. 13. Comparison of the conductivity decay in the wake of a 0.22-in.-diam nylon sphere in argon at 6 cm Hg pressure obtained with the conductivity bridge (r_c=1.38 in.) using the luminous wake width with the theoretically predicted decay assuming, (1) equilibrium, (2) Spitzer conductivity, and (3) a binary recombination process of the type $X_2^+ + e \xrightarrow{\alpha} 2X$ in combination with the luminous wake growth, Eq. (22). Note the excellent agreement assuming a binary recombination process with $\alpha=3.5\times10^{-7}$ cm³/part-sec, a reasonable value for this process (Ref. 8).

of values measured in other experiments[8] and indicates that the assumed binary recombination mechanism is plausible.

[8] S. C. Brown, *Basic Data of Plasma Physics* (Technology Press of MIT and John Wiley & Sons, Inc., New York, 1959), pp. 192–197.

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Apparatus for Measuring Characteristics of Superconducting Tunnel Junctions*

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Much information about the density of electron states in a superconductor may be obtained using electron tunneling techniques. It is of particular interest to measure the normalized dynamic conductance of superconducting tunnel junctions as a function of applied voltage. This quantity is the ratio of the dynamic conductance $(di/dv)_s$ when one or both metallic members of the tunnel junction are in the superconducting state, to the dynamic conductance $(di/dv)_n$ when both members are normal. A new method has been developed which enables measurements to be made of $(di/dv)_s/(di/dv)_n$ to a few parts in ten thousand. With this method only a bridge circuit, a galvanometer amplifier, and an oscilloscope are used. The galvanometer amplifier has a passband from dc to a few cps and an input noise voltage of about 3×10^{-8} rms V. The circuits of the bridge and amplifier are presented and analyzed. The operation of the circuits for measuring the characteristics of low resistance specimens ($\lesssim 500\Omega$) is described.

INTRODUCTION

MUCH information about the density of electron states in a superconductor may be obtained using electron tunneling techniques.[1–3] It is of particular interest to measure the normalized dynamic conductance of superconducting tunnel junctions as a function of applied voltage. This quantity is the ratio of the dynamic conductance $(di/dv)_s$ when one or both metallic members of the tunnel junction are in the superconducting state, to the dynamic conductance $(di/dv)_n$ when both members are normal. Those variations in the effective density of electron states in a superconductor which mirror the phonon spectrum can be extremely small in some metals,[2,4] but they have been described theoretically[3] and can be charted experimentally with sensitive conductance measurements. Two methods of making conductance measurements with the requisite sensitivity have been described[1,5] recently. The bridge method described here, although somewhat less direct, is much simpler to construct and is capable of yielding the normalized dynamic conductances of suitable specimens with an accuracy of a few parts in ten thousand.

A typical tunnel junction between two normal metals is not strongly nonlinear in the region of interest, since the junction resistance only decreases by a factor of two as the applied voltage is increased from 0 to about 250 mV. If such junctions are cooled so that one of the metals becomes superconducting, a strong nonlinearity develops at the origin in the *i-v* characteristic due to the energy gap in the density of electron states of the superconductor. Also, far weaker deviations from the characteristic obtained for normal metals occur at voltages corresponding to the energies of the transverse and longitudinal phonons in the metal lattice. It is these small deviations that are of specific interest here. They generally occur in the voltage range between 5 and 50 mV where the *i-v* characteristic is approximately linear. This near linearity is exploited by our bridge but, since highly nonlinear regions can be approximated as closely as one wishes by a succession of sufficiently short linear relations, the apparatus may be used, with a little extra patience, to investigate the energy gap region as well.

THE BRIDGE

Principle of the Method

In Fig. 1 a bridge is shown, one arm of which contains a nonlinear passive element R_x, having a voltage v across it. As long as the input resistance R_g of the preamplifier is large enough, the current through it will be negligible and

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[1] I. Giaever, H. R. Hart, and K. Megerle, Phys. Rev. **126**, 941 (1962).
[2] J. M. Rowell, P. W. Anderson, and D. E. Thomas, Phys. Rev. Letters **10**, 334 (1963).
[3] J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, Phys. Rev. Letters **10**, 336 (1963).
[4] J. G. Adler and J. S. Rogers, Phys. Rev. Letters **10**, 217 (1963).
[5] D. E. Thomas and J. M. Klein, Rev. Sci. Instr. **34**, 920 (1963).