

On Laminar Flame Quenching and Its Application to Spark Ignition Engines

COLIN R. FERGUSON and JAMES C. KECK

*Massachusetts Institute of Technology, Department of Mechanical Engineering, Room 3-342,
77 Massachusetts Ave., Cambridge, Mass., 02139*

Analyses are made of flame quenching in flows both perpendicular and parallel to a heat sink. A fundamental question is raised about the characteristic length over which a flame in a tube loses heat. Two possibilities are examined, the radius and the flame thickness. By using a functional form prescribed by the theoretical analyses a correlation of laminar flame speed, quenching diameter, and lean limit flame temperature is developed. The correlation suggests that the characteristic length for heat loss is the flame thickness. Laminar flame speeds of isoctane and propane have been correlated and used to compute quench distances in an engine. The agreement is reasonable and leads to speculation that quenching in an engine is basically laminar and dependent upon the free stream unburned gas temperature rather than the wall temperature.

I. INTRODUCTION

Correlations of laminar quenching distances have found application in the modeling of quench generated hydrocarbons in internal combustion engines. Daniel [1], in attempting to show that his measurements of quench distances in a CFR engine were the same order of magnitude as quench distances measured in laminar flow devices, established a precedent for the type of correlations consequently used in engine simulations (for example, see Hiroyasu and Kadota [2]). These scaling laws extrapolate dependence on unburned temperature far beyond the range of the experimental data. They also assume that the quench distance is independent of the residual burned gas in the unburned mixture. Since experimental data are lacking a physical correlation is desirable. Such correlations exist [3,4] but they have not incorporated what in the present paper, and after Mayer [5], will be called the tangency condition. A correlation based on a thermal theory including the tangency condition will be presented.

It is not the intent of this paper to suggest that heat loss alone quenches a flame. It is well known that the important features of flame quenching can also be illustrated by assuming that chain breaking at the wall quenches a flame [6,7] and that there are phenomena that a thermal theory can not explain [8]. One purpose of the present paper is, however, to develop a correlation of flame speed and quenching distance. Towards that goal a thermal theory is applicable.

In the following sections, analyses are made of quenching in flows both perpendicular and parallel to heat sinks. A correlation of laminar flame speed, quenching diameter, and lean limit adiabatic flame temperature is presented. Finally application of the correlation is illustrated by calculation of quench distances in an engine.

II. LAMINAR FLAME QUENCHING

Flow Perpendicular to a Wall

Consider the steady flame which can be stabilized on a porous flat flame burner as depicted in Fig. 1. The

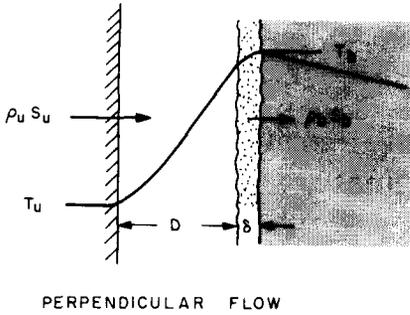


Fig. 1. Flame quenching in a perpendicular flow.

mass flow rate per unit area is the product of the unburned density ρ_u and the laminar flame speed S_u . The first law of thermodynamics applied to the system bounded by the burner surface and the plane of maximum temperature reads

$$\rho_u S_u c_{pb} (T_b^\circ - T_b) = k \frac{(T_b - T_u)}{D}, \quad (1)$$

where T_b° —adiabatic flame temperature, T_b —maximum flame temperature, c_{pb} —specific heat of burned gases, k —thermal conductivity, D —distance of flame from wall, T_u —temperature of unburned mixture and the heat sink.

Implicit in the notation is the fact that for adiabatic combustion ($D \rightarrow \infty$) the enthalpies of reactants and products are equal and $T_b = T_b^\circ$. If the burned gas temperature and flame speed are known one can predict the distance the flame will be from the burner.

The scaling of heat loss from a flame, the right hand side of Eq. (1), depends upon the distribution of temperature along the spatial coordinate (likewise a similar analysis for flow parallel to a heat sink will show a dependence upon the distribution of heat loss in the flame). By choosing dimensionally correct parameters for the scaling, one defines a constant of proportionality. It will be assumed that the thermal conductivity characteristic of the flame is calculated from the reactant composition but at the burned gas temperature. This is consistent with describing the flame thickness as dominated by a preheat zone of negligible chemical reaction. The constant of proportionality

so defined is assumed to be of order unity and hence ignored theoretically.

Kaskan [9] has shown that the flame speed on flat plate burners often correlates empirically as

$$\frac{S_u}{S_u^\circ} = \exp\left(\frac{-E}{2R} \left(\frac{1}{T_b} - \frac{1}{T_b^\circ}\right)\right), \quad (2)$$

where E is an apparent activation energy and R is the gas constant. Substitution into Eq. (1) and rearrangement into dimensionless groups yields a Peclet number

$$\frac{\rho_u S_u^\circ c_{pb} D}{k} = \frac{T_b - T_u}{T_b^\circ - T_b} \exp\left(\frac{-E}{2R} \left(\frac{1}{T_b} - \frac{1}{T_b^\circ}\right)\right). \quad (3)$$

Solution of Eq. (3) is illustrated in Fig. (2) for the particular case of propane-air combustion with the unburned mixture at atmospheric density. The right-hand side of Eq. (3) is a U-shaped function of temperature. The left-hand side of Eq. (3) is

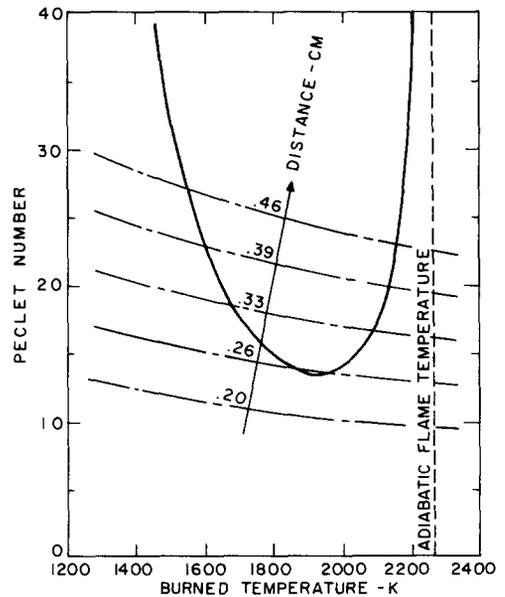


Fig. 2. Solution of the quenching equation for perpendicular flow of stoichiometric propane-air. $P = 1$ atm, $T_u = 298$ °K, $T_b^\circ = 2268$ °K, $S_u^\circ = 36$ cm/sec [24], $c_{pb} = .53$ cal/gK, $E = 53$ kcal/gmole [9].

plotted as lines of constant distance D from the wall. It can be seen that for sufficiently large distances from the wall there is both a high-temperature and a low-temperature solution. There is also a critical distance, in this case $D_1 = .26$ cm, such that if $D < D_1$ then no solution exists. When a quasi-steady assumption is valid this critical distance is the quenching distance for an unsteady flame propagating into a wall.

As shown in Fig. 2, the quenching distance is determined by a tangency condition.

$$\frac{\partial}{\partial T_b} \left(\frac{\rho_u S_u c_{pb} D}{k} \right) = \frac{\partial}{\partial T_b} \left(\frac{T_b - T_u}{T_b^\circ - T_b} \right) \times \exp \left(\frac{-E}{2R} \left(\frac{1}{T_b^\circ} - \frac{1}{T_b} \right) \right) \quad (4)$$

There are two solutions to Eq. (4) in the domain of interest, i.e., $T_u \leq T_b \leq T_b^\circ$. The low-temperature solution is an artifact of the scaling law for laminar flame speed. The problem is reminiscent of the cold-boundary difficulty encountered in solving flame equations in that the flame speed does not go to zero as $T_b \rightarrow T_u$. Consequently the Peclet number goes to zero as $T_b \rightarrow T_u$ and there is a local maximum in the U-shaped curve.

The solution of interest is the high-temperature solution depicted in Fig. 2. For large activation energy it can be shown that the first two terms of an asymptotic series for the critical temperature (the temperature at quenching) are

$$T_b^* \sim T_b^\circ \left(1 - \frac{2RT_b^\circ}{E} \right), \quad (5)$$

and that the critical Peclet number is asymptotic to

$$Pe^* \sim e \frac{E}{2RT_b^\circ} \left(\frac{T_b^\circ - T_u}{T_b^\circ} \right), \quad (6)$$

where $e = 2.72$.

Flow Parallel to a Wall

The problem to be analyzed is shown in Fig. 3. A coordinate system is selected so that the flame is

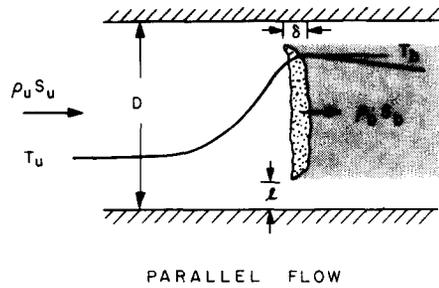


Fig. 3. Flame quenching in a parallel flow.

stationary. Hence the walls are moving with the laminar flame speed. Downstream of the flame, in the burned zone, the gases are moving at a speed different than the wall so that somewhere in the flame a hydrodynamic boundary layer begins to grow. Likewise the walls are assumed to be at temperature T_u so that a thermal boundary layer grows. The situation is more complicated than for flow perpendicular to a wall. There are solutions to the two-dimensional problem [10,11] but in each case an ignition temperature must be introduced into the analysis.

The present theory, which may be called quasi-one-dimensional, assumes that the aforementioned boundary layers occupy a negligible fraction of the control volume drawn for the energy analysis. The system to be analyzed is bounded by the walls and by planes drawn immediately upstream and downstream of the flame. The characteristic length over which the flame loses heat is assumed to be the dead space of thickness l . For flow in a tube the energy balance yields

$$\rho_u S_u c_{pb} (T_b^\circ - T_b) \frac{\pi}{4} (D - 2l)^2 = k \frac{(T_b - T_u)}{l} \delta \pi D, \quad (7)$$

where δ is a length characteristic of the flame thickness.

Experimentally it is observed that the dead space thickness l is small compared to the quenching distance [10], so that Eq. (7) may be re-

arranged to read

$$\frac{\rho_u S_u^\circ c_{pb} D}{k} \cong G \frac{\delta}{l} \frac{T_b - T_u}{T_b^\circ - T_b} \times \exp\left(\frac{-E}{2R}\left(\frac{1}{T_b^\circ} - \frac{1}{T_b}\right)\right). \quad (8)$$

For tubes, $G = 4$, whereas a similar development for parallel plates yields $G = 2$.

The scaling of δ/l is unknown. On the basis of experimental evidence presented later it will be assumed that δ/l is a constant of order unity. In that case the tangency condition becomes identical to that solved for perpendicular flow and the critical Peclet number is asymptotic to

$$\text{Pe}^* \sim G e \frac{E}{2RT_b^\circ} \left(\frac{T_b^\circ - T_u}{T_b^\circ}\right). \quad (9)$$

Alternatively Mayer and others [12-14] have assumed that the characteristic length over which the flame loses heat is half the tube diameter (or plate separation). Introducing a suitable scaling for δ , Eq. (8) becomes

$$\left(\frac{\rho_u S_u^\circ c_{pb} D}{k}\right)^2 = 2G \frac{T_b - T_u}{T_b^\circ - T_b} \times \exp\left(\frac{-E}{R}\left(\frac{1}{T_b^\circ} - \frac{1}{T_b}\right)\right), \quad (10)$$

which is essentially Mayer's result. For a scaling like Eq. (10), the asymptotic solution to the tangency condition is

$$T_b^* \sim T_b^\circ \left(1 - \frac{RT_b^\circ}{E}\right), \quad (11)$$

and the critical Peclet number is asymptotic to

$$\text{Pe}^* \sim \left(2G e \frac{E}{RT_b^\circ} \frac{T_b^\circ - T_u}{T_b^\circ}\right)^{1/2}. \quad (12)$$

Empirical Correlation

Most of the available quenching data are for parallel flow in tubes or between parallel plates. The authors could find no data for the steady flow perpendicular to a wall and only one data set for unsteady flow perpendicular to a wall [15]. Therefore the correlation will be developed for parallel flow. On the basis of the previous analysis the correlation will be of the form

$$\frac{\rho_u S_u^\circ c_{pb} D_0}{k_u(T_b^\circ)} = F \left(\frac{E}{RT_b^\circ} \frac{T_b^\circ - T_u}{T_b^\circ}\right)^n, \quad (13)$$

where the notation $k_u(T_b^\circ)$ indicates that the thermal conductivity is to be computed using the unburned gas composition and the adiabatic flame temperature.

The determination of F and n from experimental data is simple in principle but complicated by the following facts: (1) there are considerable discrepancies in the literature on measured flame speeds, (2) there is little overlap of flame speed data and quenching data and (3), the authors could find no data from which the apparent activation energy for parallel flows could be determined unambiguously. To overcome the above (1) and (3), a set of selection rules was employed to choose flame speed data and the apparent activation energy E was assumed to scale with the lean limit flame temperature T_{bL}° .

The selection rules were based on a survey of burning velocity by Andrews and Bradley [16]. Data sets that reported flame speeds within 5% of their recommended values were considered acceptable for that fuel and oxidant. Where no recommended values were given, as for propane, experimental methods employing the double kernel method, a particle tracking technique, or measurements made in closed vessel explosions were considered acceptable.

The correlation of activation energy with the lean limit flame temperature was suggested by Fenn [17]. The basic argument may be illustrated by solving Eq. (5) for the activation energy

$$E = 2RT_b^\circ \left(\frac{T_b^\circ - T_b^*}{T_b^\circ}\right). \quad (14)$$

TABLE 1

The Following Matrix Illustrates the Data Sets Used in Fig. 4

	ϕ	T_u (°K)	P (atm)	Refs.
C ₂ H ₄ + Air	.7-1.4	298	1.0	[21, 22]
C ₃ H ₈ + Air	.8-1.3	270-497	.56-2.25	[21, 22, 23, 24]
H ₂ + Air	1.0-2.0	313	1.0	[25, 4]
CH ₄ + Air	.6-1.4	298	1.0	[26, 27]
CH ₄ + O ₂	.4-2.0	298	1.0	[27, 28]
H ₂ + O ₂	1.0	298	1.0	[28, 29]

By assuming that lean limit measurements made in tubes are the results of flame quenching and that the nondimensional temperature defect $(T_b^\circ - T_b^*)/T_b^\circ$ is invariant at the lean limit, the activation energy is determined.

The resultant data to be correlated are described by Table 1. Thermochemical and transport calculations were done with the computer program of Svehla and McBride [18]. Lean limit compositions were taken from the report by Coward and Jones [19]. In some cases quenching distances between parallel plates were converted to quenching diameters by the formula $D_{||} = .65D_0$ [20].

In examining the propane-air data, it was found that using the frozen specific heat yielded (at constant pressure) a single curve independent of equivalence ratio whereas the equilibrium specific heat produced families of curves. Consequently all further correlation was done using frozen specific heats.

Least squares analyses were performed on the data set and it was determined that the constants n and F (redefined to incorporate the constant of proportionality between lean limit flame temperature and activation energy) were bounded by

$$0.86 \leq n \leq 1.05,$$

$$23.8 \leq F \leq 26.8. \tag{15}$$

Herein lies the experimental support of Eq. (9) rather than Eq. (12), and on that basis it was decided that $n = 1.0$ and $F = 25$. The magnitude of F justifies *a posteriori* the assumption that constants of proportionality in the theoretical anal-

yses are of order unity. Furthermore, from experimental data [15,20] the constants F for parallel plates and perpendicular flow would be 16 and 6.5, respectively, whereas theoretically they should be 12 and 6.2.

The goodness of the fit is displayed in Fig. 4. Nearly two orders of magnitude of quenching diameters have been correlated. The standard deviation of the fit is approximately 20% with no error greater than 50%.

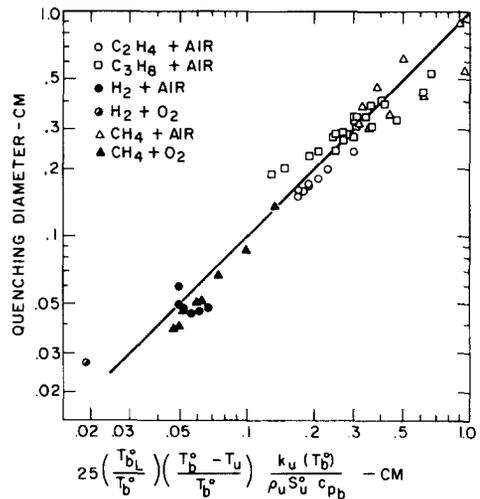


Fig. 4. Correlation of quenching diameter with laminar flame speed and lean limit flame temperature. The thermal conductivity $k_u(T_b^0)$ is evaluated using the reactant composition and the adiabatic flame temperature. Likewise the frozen specific heat of the burned gases c_{pb} is evaluated at the adiabatic flame temperature.

III. APPLICATION TO SPARK IGNITION ENGINES

The modeling of flame quenching in an engine generally requires the computation of quenching distance many times. For this reason it is desirable to use approximate methods for the computation of thermodynamic and transport properties. Therefore in the present computation of the quench distances measured by Daniel [1] suitable approximations are used and will be described briefly.

The thermodynamic state of burned gases is computed using the model of Martin and Heywood [30]. From the algorithms presented in that paper it is clear how to compute the frozen specific heat of burned gas. The state of the unburned gas is computed according to equations cited by Hires et al. [31] together with appropriate polynomials to describe the fuel's enthalpy [32]. The frozen thermal conductivity based on the unburned gas composition at the adiabatic flame temperature is computed by the mixing rules given by Brokaw [33]. The potential parameters used are those of Svehla [34] and neglect the fact that water is a polar molecule (an acceptable approximation since water in the unburned mixture, introduced by residual burned gas, is a minor species). The collision integrals are computed by the formula (due in part to Westenberg [35]):

$$\Omega^{(2,2)}(T^*) = \text{MAX} (1.61T^{*-0.45}, 1.22T^{*-0.16}). \quad (16)$$

Equation (16) is within 3% of the exact answers tabulated by Hirschfelder et al. [36] over the range of reduced temperature $0.3 < T^* < 400$.

Just as it is useful to approximate the thermodynamic and transport properties, it is desirable to have a relatively simple correlation formula for laminar flame speeds. In principle such a formula should be used only for interpolation purposes; however, since there is a general lack of high pressure and high temperature flame speed data, extrapolation is often necessary (particularly with respect to equivalence ratio). Consequently it is desirable to introduce some

physics into the correlation, which generally conflicts with the desire for simplicity. A reasonable compromise is given by Van Tiggelen [37] and will be employed as representative of the state of the art.

According to Van Tiggelen's model the flame speed is

$$S_u^\circ = KC_m \left[Y_F^a Y_{O_2}^b \exp \left(-\frac{E}{RT_m} \right) \right]^{1/2}, \quad (17)$$

where $C_m = \sqrt{8RT_m/\pi M_R}$ is the mean molecular speed of chain carriers whose mean molecular weight is M_R , Y_F = mole fraction of fuel molecules in unburned mixture, Y_{O_2} = mole fraction of oxygen molecules in unburned mixture, a = reaction order with respect to fuel, b = reaction order with respect to oxygen, E = activation energy, $T_m = T_u + 0.74(T_b^\circ - T_u)$ is the mean temperature of the reaction zone [38].

Equation (17) has been applied to many fuels and oxidants [39] and it has been found for all hydrocarbons examined that $a + b = 1$, a result that will be employed.

Theoretically the dimensionless parameter K is independent of pressure. The experimental support for that conclusion is limited to the experiments of De Soete and Brasslet [40]. In order to introduce some pressure dependence empirically into the correlation it will be assumed that

$$K = \frac{2T_u}{\sqrt{3\pi}T_m} \left(\frac{P}{P_r} \right)^\alpha, \quad (18)$$

where P_r is equal to 1 atm.

Hence there are four constants to be determined from experimental data; E , M_R , α , and either a or b . These parameters have been estimated for iso-octane and propane and are given in Table 2. That the parameters are estimates is to be emphasized, as they have been determined from conflicting experimental data. The standard deviation of the fit is about 15% with no error greater than 50%.

In order to demonstrate that all the approxi-

TABLE 2
Kinetic Parameters of Isooctane and Propane

	C ₈ H ₁₈	C ₃ H ₈
M_R	68	31
α	-.22	-.06
a	-.71	-.46
b	1.71	1.46
E (kcal/mole)	38.6	37.7
T_{bL} (°K)	1825	1600
Data Refs.: [21, 24, 40, 41 ^a , 42, 19]		

^a This data set did not meet the selection rules described in the text but had to be used to estimate b for isooctane.

mations made thus far are indeed reasonable, quenching distances have been calculated for the parallel plate geometry and compared with experimental measurements. Figure 5 shows the results for the quench distances of propane and isooctane as a function of equivalence ratio.

One feature of flame quenching that the cor-

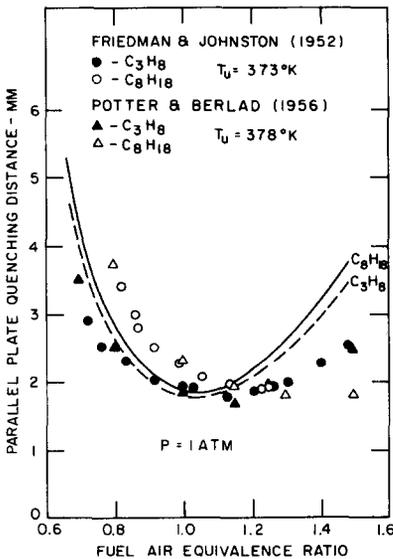


Fig. 5. Quench distances computed by coupling a flame speed correlation to the Peclet number correlation compared with experimental data. The computations were done for $T_u = 373^\circ\text{K}$.

relation is not able to account for is the fact that the minimum quenching distance does not necessarily occur at the maximum flame temperature. This is clearly shown in Fig. 5, at least for isooctane. It has been suggested by Potter and Berlad [4] that instabilities accelerate the reaction rate of some rich hydrocarbon-air flames and cause a corresponding reduction in the quenching distance. The instabilities create cells which are approximately the same size as the quenching distance (see Friedman [43]). This may or may not be a feature of quenching in an engine. There is evidence that in turbulent mixtures the preferential diffusion that causes the cells is modified by velocity fluctuations so that quenching distances exhibit minima at the equivalence ratios of maximum flame temperature [44]. In any case over the range of equivalence ratios of interest in engine combustion these effects may be ignored.

The quench distances measured by Daniel [1] can now be computed. The results are shown in Table 3 and show that indeed the quench distance in an engine scales with a laminar quench distance. The explanation may lie in the fact that viscosity enforces at the wall a no-slip condition, the result of which is a viscous sublayer. Computations of the characteristic viscous length may be made by estimating the fluctuating velocity u characteristic of the field far from the wall. A Reynolds number

TABLE 3
Dead Space in a Propane-Fueled Engine and Predicted Laminar Quench Distance for Perpendicular Flow ($F = 6.5$)^a

p	$\bar{T}_{u\infty}$	ϕ	Y_R	l (Daniel [1])	D_{\perp}
	°K			μm	μm
13.4	610.	1.57	.07	102	91
15.0	680.	0.97	.07	51	33
13.4	670.	0.71	.08	76	66
4.0	680.	1.02	.20	381	172

^a The free stream unburned gas temperature $\bar{T}_{u\infty}$ and residual mole fraction Y_R were estimated from an engine simulation [46] which matched Daniel's combustion intervals. The inlet temperature was assumed to be 311°K .

defined as

$$\text{Re} = \frac{uD_{\perp}}{\nu}, \quad (19)$$

where ν is the kinematic viscosity, may be interpreted as the ratio of the quench distance to the viscous length. The computation of that Reynolds number is not straightforward. If one bases the viscosity on the average temperature of the gas in the layer of thickness D_{\perp} prior to the arrival of the flame the Reynolds number is of the order of 100. However, if one chooses an average temperature subsequent to quench, then the number is of the order of 10 and low enough to neglect the effects of turbulence on the quenching.

One other feature of the computation has yet to be discussed. The correlation of laminar quenching distances assumed that the unburned mixture and the heat sink were at the same temperature. In an engine this is not the case since the unburned mixture temperature is time dependent whereas the walls are practically at a constant temperature. The result is that there exists at the wall a thermal boundary layer into which the flame propagates while quenching. As Table 3 shows it is a reasonable approximation to assume that quenching occurs at the average temperature of unburned mixture in the whole of the cylinder. One can only speculate that since the quenching Reynolds number, Eq. (19), is of the order of 100 prior to quenching most of the gas about to participate in the quenching is very nearly at the unburned mixture temperature. This is because the viscous sublayer, which is only ten viscous lengths in thickness is responsible for most of the temperature drop in a turbulent boundary layer. This description is supported by the fact that Gottenberg et al. [45] found the quench generated hydrocarbons in a bomb (simulating engine combustion) were independent of wall temperature.

IV. CONCLUSIONS

Nearly two orders of magnitude of quenching diameter and flame speeds have been correlated by a Peclet number. The Peclet number is based on the quench distance, flame speed, unburned gas

density, frozen specific heat of the burned gas, and frozen thermal conductivity of the unburned gas at the adiabatic flame temperature. The Peclet number is correlated with the flame temperature and the lean limit flame temperature by

$$\text{Pe} = F \frac{T_{bL}^{\circ}}{T_b^{\circ}} \left(\frac{T_b^{\circ} - T_u}{T_b^{\circ}} \right), \quad (20)$$

where $F = 25, 16,$ and 6.5 for quenching diameter, parallel-plate quenching distance, and perpendicular quench distance, respectively.

Suitable approximations have been described for applying the Peclet number correlation to the computation of flame quenching in an engine. The quench distance in an engine scales with a laminar quench distance computed from the state of the unburned gas at the time of quench. The reason for this behavior may be explained by the existence of a viscous sublayer.

REFERENCES

1. Daniel, W. A., *Sixth Symposium (International) on Combustion*, Reinhold, New York, 1957, p. 886.
2. Hiroyasu, H., and Kadota, T., *Fifteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburg, 1975, p. 1213.
3. Brokaw, R. S., and Gerstein, M., *Sixth Symposium (International) on Combustion*, Reinhold, New York, 1957, p. 66.
4. Potter, A. E., and Berlad, A. L., *Sixth Symposium (International) on Combustion*, Reinhold, New York, 1957, p. 26.
5. Mayer, E., *Combust. Flame* **1**, 438-452 (1957).
6. Simon, D. M., Belles, F. E., and Spakowski, A. E., *Fourth Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1953, p. 126.
7. De Soete, G., and Van Tiggelen, A., *VDI-Berichte* Nr 146, 35-43 (1970).
8. Tewari, G. P., and Weinberg, F. J., *Proc. Roy. Soc. A* **296**, 546-565 (1966).
9. Kaskan, W. E., *Sixth Symposium (International) on Combustion*, Reinhold, New York, 1957, p. 134.
10. von Kármán, T., and Millán, *Fourth Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1953, p. 126.
11. von Elbe, G., and Lewis, B., *Third Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1949, p. 68.

12. Spalding, D. B., *Proc. Roy. Soc. A* **240**, 83-100 (1957).
13. Berlad, A. L., and Yang, C. H., *Combust. Flame* **4**, 325-333 (1960).
14. Gerstein, M., and Stine, W. B., *Fourteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1973, p. 1109.
15. Ellenberger, J. M. and Bowlus, D. A., 1971 Technical Session, Central States Section, The Combustion Institute (1971).
16. Andrews, G. E., and Bradley, D., *Combust. Flame* **18**, 133-153 (1972).
17. Fenn, F. B., and Calcote, H. F., *Fourth Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1953, p. 231.
18. Svehla, R. A., and McBride, B. J., NASA TN D-7056 (1973).
19. Coward, H. F., and Jones, G. W., Bull. 503, Bureau of Mines (1952).
20. Berlad, A. L., and Potter, A. E., *Fifth Symposium (International) on Combustion*, Reinhold, New York, 1955, p. 728.
21. Raezer, S. D., and Olsen, H. L., *Combust. Flame* **6**, 227-232 (1962).
22. Wilson, C. W., *Ind. Eng. Chem.* **51**, 560-563 (1959).
23. Friedman, R., and Johnston, W. C., *J. App. Physics* **21**, 791-795 (1950).
24. Chase, J. D., and Weinberg, F. J., *Proc. Roy. Soc. A* **275**, 411-430 (1963).
25. Heimel, S., NACA TN 4156 (1957).
26. Andrews, G. E., and Bradley, D., *Combust. Flame* **18**, 275-288 (1972).
27. Harris, M. E., Grumer, J., von Elbe, G., and Lewis, B., *Third Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1949, p. 80.
28. Strauss, W. A., and Edse, R., *Seventh Symposium (International) on Combustion*, Butterworths, London, 1959, p. 377.
29. Friedman, R., *Third Symposium (International) on Combustion*, Williams & Wilkins, Baltimore, 1949, p. 110.
30. Martin, M. K., and Heywood, J. B., to appear in *Combust. Sci. & Tech.*
31. Hires, S. D., Ekchian, A., Heywood, J. B., Tabaczynski, R. J., and Wall, J. C., SAE #760161 (1976).
32. LoRusso, J., S. M. Thesis, Dept. of Mech. Eng., M.I.T. (1976).
33. Brokaw, R. S., NASA TN R-81 (1961).
34. Svehla, R. A., NASA TR R-132 (1962).
35. Westenberg, A. A., *Combust. Flame* **1**, 346-359 (1957).
36. Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B., *Molecular Theory of Gases and Liquids*, John Wiley & Sons, Inc., 1954.
37. Van Tiggelen, A., and Deckers, J., *Sixth Symposium (International) on Combustion*, Reinhold, New York, 1957, p. 61.
38. Van Tiggelen, P. J., and Duval, A., *Bulletins de L'Academie Royal de Belgique—Classes des Sciences* **8**, 326-365 (1967).
39. Van Tiggelen, A., et collaborateurs, *Oxydations et Combustions*, Editions Technip, Paris, 1968, TOME 1, p. 536.
40. De Soete, G., et Brasslet, J., *Revue de L'Institut Francais au Petrole XXIV*, No. 12, 1969, p. 1602.
41. Gibbs, G. J., and Calcote, H. F., *J. Chem. Eng. Data* **4**, 226-237 (1959).
42. Babkin, V. S., V'yum, A. V., and Kozachenko, L. S., *Combust. Explosion, and Shock Waves* **3**, 221-225 (1967).
43. Friedman, R., *J. Aero. Sci* **18**, 499 (1951).
44. Ballal, D. R., and Lefebvre, A. H., *Fifteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1974, pp. 1473-1481.
45. Gottenberg, W. G., Olson, D. R., and Best, H. W., *Combust. Flame* **7**, 9-16 (1963).
46. Tabaczynski, R. J., Heywood, J. B., DOT Report No. 00, Vol. 5 (June 1976).

Received 6 May 1976; revised 5 August 1976