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IT WAS FIRST shown by Daniel (1*,2), that the primary source of unburned hydrocarbons (HC) from spark ignition engines burning premixed charges is the cool quench layer on the combustion chamber walls. In subsequent experiments Tabaczynski, Heywood and Keck (3) showed that in an unthrottled engine the unburned HC leave the combustion chamber in two distinct peaks at the beginning and end of the exhaust process. The first peak is due to entrainment of the head wall quench layer during the initial blowdown of the cylinder; the second peak is due to entrainment of the side wall vortex formed by "roll up" of the side wall quench layer by the piston during the exhaust stroke.

The existence of the side wall vortex and its size have been established in experiments using a water analog by Tabaczynski, Hoult and Keck (4). These results permit one to estimate the contribution of unburned HC from the side wall vortex as a function of engine geometry and operating conditions.

The purpose of the present paper is to present a model of quench layer entrainment which

*Numbers in parentheses designate References at end of paper.

ABSTRACT

An aerodynamic model of the entrainment of the head wall quench layer during blowdown and exhaust of an internal combustion engine has been developed. The model may be used to calculate the time resolved concentration and mass flowrate of hydrocarbons (HC) in the exhaust, from a knowledge of engine geometry and operating conditions. It predicts that the area A_s from which HC are swept will be proportional to the cube root of the ratio of the quench layer thickness δ_q to the thickness of the viscous boundary layer δ_v. Since the mass of HC emitted is proportional to the product of the HC density ρ_{HC}, the area A_s and the thickness δ_q, the HC emissions will be proportional to the product $\rho_{HC}\ \delta_q^{4/3}$ and this is the most important factor determining the emissions.

The model also predicts that the time dependence of the HC mass flowrate will depend relatively strongly on the pressure ratio p_4/p_e across the exhaust valve when it opens: in an unthrottled engine virtually all the head wall HC exit during blowdown; in a heavily throttled engine the HC are emitted more or less uniformly during the exhaust stroke.

It is expected that the model will be useful both for interpreting experimental measurements of HC emissions and for predicting HC emissions from practical engines. Comparisons of the results obtained to date with the available experimental data shows good agreement.

will permit corresponding estimates of the unburned HC from the head wall to be made.

A general aerodynamic model of quench layer entrainment by flow into a line sink is described in the following section. Application of the model to the prediction of the time resolved HC emissions from internal combustion engines and a comparison with the experimental results of Ref. (3) are made in the third section. A brief summary and conclusions are contained in the final section.

FLOW INTO A LINE SINK

TOTAL MASS FLOW INTO SINK - The flow of exhaust gases within the combustion chamber is modelled by the two dimensional boundary layer flow into a line sink shown in Fig. 1. The velocity at any point external to the boundary layer, u_e, is inversely proportional to the radial distance from the sink r_e, ie:

$$u_{e,1}r_{e,1} = u_{e,2}r_{e,2} = u_e r_e \qquad (1)$$

so that, provided the kinematic viscosity ν is everywhere constant, the Reynolds number defined as

$$Re = \frac{u_e r_e}{\nu} \qquad (2)$$

is invariant throughout the external flow field.

Schlichting (5) has presented solutions to the Navier-Stokes equations for sink flow in which the velocity distribution within the boundary layer is related to the external velocity by:

$$u = u_e f'(\eta_v) \qquad (3)$$

where

$$f'(\eta_v) = 3 \tanh^2(\frac{\eta_v}{\sqrt{2}} + 1.146) - 2 \qquad (4)$$

and the coordinate η_v is defined as

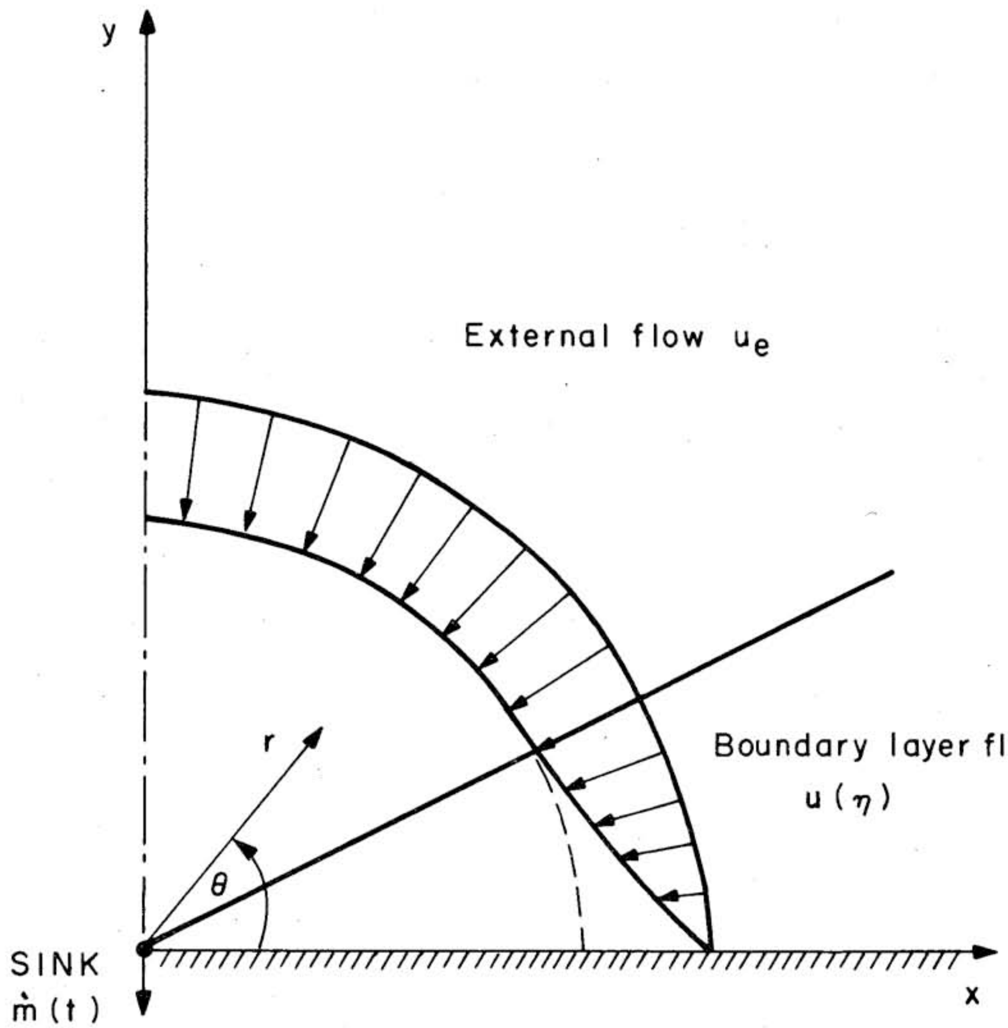

Fig. 1 - Schematic diagram of flow into a line sink.

$$\eta_v = \frac{y}{x} Re^{\frac{1}{2}} = Re^{\frac{1}{2}}\tan\theta \approx Re^{\frac{1}{2}}\theta \qquad (5)$$

Thus the velocity distribution throughout the entire field of flow can be determined in terms of some reference flow rate, which for convenience may be taken to be the sink strength, ie, the mass flow rate into the sink. Neglecting hold up in the boundary layer the sink strength is:

$$\dot{m}(t) = \ell \int_0^{\pi/2} \rho(t) u_e r_e d\theta$$

or (6)

$$\dot{m}(t) = (\tfrac{\pi}{2}) \ell \rho(t) \nu Re$$

where: $\dot{m}(t)$ = inviscid sink strength at time t,
$\rho(t)$ = gas density - assumed constant throughout the combustion chamber,
ℓ = length of the sink.

Eqs. (2), (3), (5) and (6) relate the flow in the exhaust port to the velocity distribution in the wall regions of the combustion chamber.

MASS FLOW OF SPECIES α INTO SINK - The problem of determining the time resolved exhaust rate of a species α, having a known initial spatial distribution within the combustion chamber, is simply one of determining the origins of the fluid particles entering the line sink at time t. If the curved line in Fig. 2 represents the locus of such a set of origins, then the cross-hatched area represents the total mass of exhaust gases that has entered the sink t seconds after the exhaust valve has opened. The mass of species α that has entered the sink is then

$$m_\alpha(t) = \ell \int_0^{r_e} \int_\theta^{\pi/2} \rho_\alpha r d\theta' dr \qquad (7)$$

in which ρ_α is the density distribution of species α and may be represented by

$$\rho_\alpha(t,y) = \rho_\alpha(t) k_\alpha(\eta_\alpha)$$

with (8)

$$k_\alpha(0) = 1$$

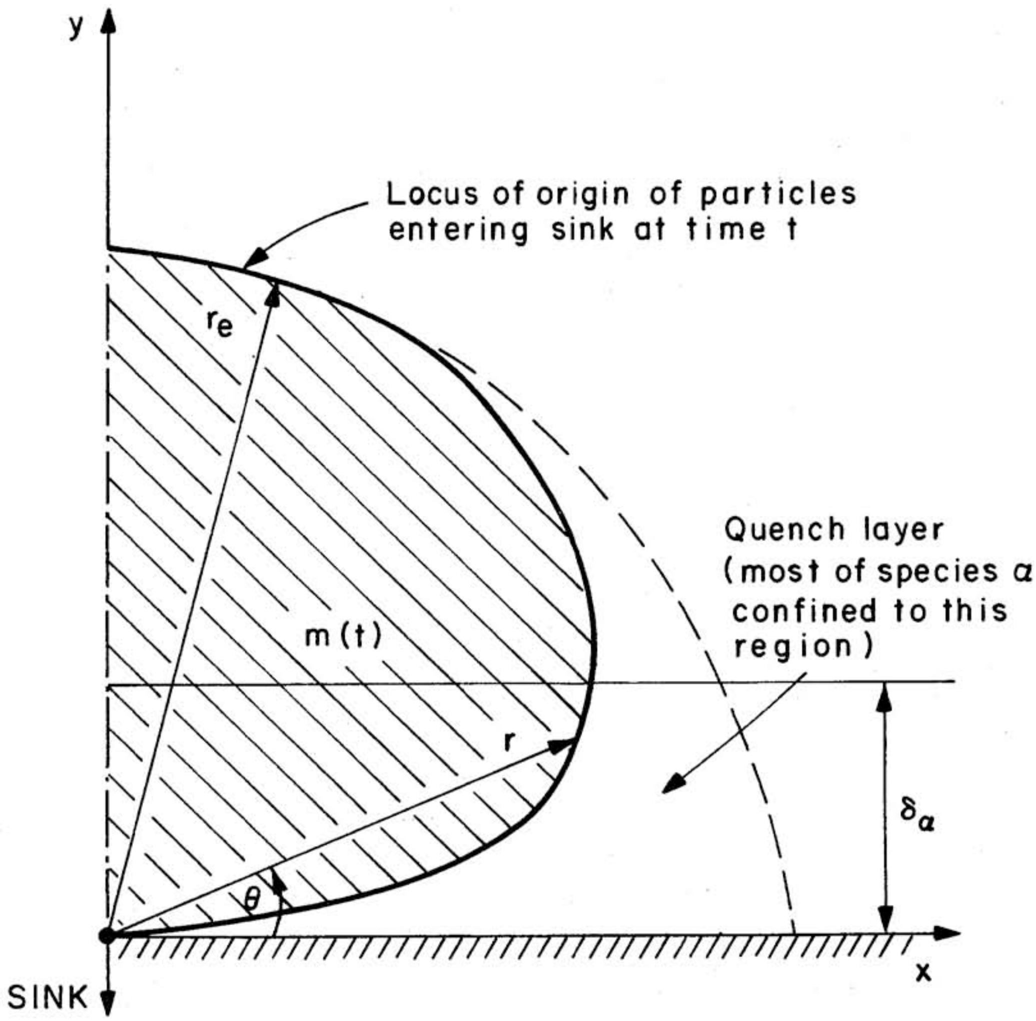

Fig. 2 - Schematic diagram showing gas exhausted up to time t.

The coordinate η_α is defined as

$$\eta_\alpha = \frac{y}{\delta_\alpha} = \frac{r\sin\theta}{\delta_\alpha} \quad (9)$$

δ_α is a length characteristic of the species under consideration; eg. for unburned hydrocarbons δ_{HC} would be the quench layer thickness. In this case of unburned hydrocarbons the region of interest is in the immediate vicinity of the wall, ie. θ small, so that Eq. (9) may be written:

$$\eta_\alpha = \frac{r\theta}{\delta_\alpha} \qquad (\theta \text{ small}) \quad (10)$$

and further,

$$k_\alpha(\eta_\alpha) << 1 \qquad (\eta_\alpha > 1) \quad (11)$$

Differentiation of Eq. (9) yields:

$$d\theta = \frac{\delta_\alpha d\eta_\alpha}{r\cos\theta} = \frac{\delta_\alpha d\eta_\alpha}{(1 - \delta_\alpha^2\eta^2/r^2)^{\frac{1}{2}}} \quad (12)$$

which, upon substitution into Eq. (7) results in a species mass flow given by:

$$m_\alpha(t) = \ell\rho_\alpha\delta_\alpha \int_0^{r_e}\int_{y/\delta_\alpha}^{r/\delta_\alpha} \frac{k_\alpha(\eta)d\eta dr}{(1 - \delta_\alpha^2\eta^2/r^2)^{\frac{1}{2}}} \quad (13)$$

Applying the condition expressed in Eq. (11) and the further condition that $r >> \delta_\alpha$, it is possible to simplify Eq. (13):

$$m_\alpha(t) \approx \rho_\alpha\delta_\alpha\ell r_e \int_0^1 \int_{\eta_\alpha}^\infty k_\alpha(\eta)d\eta d\tilde{r} \quad (14)$$

in which

$$\tilde{r} = r/r_e \quad (15)$$

It is now meaningful to introduce the concept of a "swept length":

$$r_s(t) = r_e(t) \int_0^1 \int_{\eta_\alpha}^\infty k_\alpha(\eta)d\eta d\tilde{r} \quad (16)$$

so that Eq. (14) may be rewritten:

$$m_\alpha(t) = \rho_\alpha\delta_\alpha\ell r_s(t) \quad (17)$$

The problem of evaluating $m_\alpha(t)$ has thus been reduced to one of determining the swept length $r_s(t)$. However, to determine $r_s(t)$ by means of Eq. (16) it is necessary to first derive a relation between η_α and $\tilde{r}$; this is equivalent to determining the shape of the bounds of the cross-hatched region in Fig. 2.

DETERMINATION OF SWEPT LENGTH - The velocity u_r at any point r is given by:

$$\frac{dr}{dt} = u_r = \frac{u_r}{u_{e,r}} \cdot \frac{u_{e,r}r}{r}$$

Substitution of Eqs. (1)-(3) yields:

$$rdr = f'(\eta_v)\nu Re\, dt$$

which on integration becomes

$$r^2 = 2\int_0^t f'(\eta_v)\nu Re\, dt' \quad (18)$$

This equation can be evaluated numerically for any given flow (Reynolds number) - time behavior to obtain a relation between r and η_v. However, an approximate analytic procedure is pursued here in the interests of obtaining a more complete physical picture of the exhaust process.

For $\eta_v << 1, f'(\eta_v) \approx f''(o) \cdot \eta_v$ so that Eq. (18) becomes, using Eq. (5):

$$r^2 = 2f''(o)\theta \int_o^t Re^{3/2}\nu dt' \qquad (\theta \text{ small}) \quad (18a)$$

while for $\eta_v >> 1, f'(\eta) = 1$ and $r^2 \approx r_e^2$ so that Eq. (18) can be written:

$$r_e^2 = 2\int_o^t Re\nu dt' \qquad (\theta \text{ large}) \quad (18b)$$

By assuming an exponential growth of r^2 with θ, asymptotic to the two extreme values given in Eqs. (18a) and (18b), we can write:

$$\frac{r^2}{r_e^2} = \tilde{r}^2 \approx (1 - e^{-\theta/\theta_o}) \quad (19)$$

where

$$\theta_o = \frac{r_e^2}{2f''(o)\int_o^t Re^{3/2}dt'} \quad (20)$$

Inverting Eq. (19) gives an expression for the polar angle θ:

$$\theta = -\theta_o \ln(1 - \tilde{r}^2) \quad (21)$$

which upon substitution into Eq. (10) yields the desired relationship between η_α and $\tilde{r}$:

$$\eta_\alpha = -\beta\tilde{r}\ln(1 - \tilde{r}^2) \quad (22)$$

with

$$\beta = \frac{r_e^3}{2f''(o)\delta_\alpha\int_o^t Re^{3/2}\nu dt'} \quad (23)$$

Substitution of Eq. (23) into Eq. (16) results in a slightly different expression for the swept length:

$$r_s = [2f''(o)\delta_\alpha \int_o^t Re^{3/2}\, dt']^{1/3} I(\beta) \qquad (24)$$

with

$$I(\beta) = \beta^{1/3} \int_o^1 \int_{\eta_\alpha}^{\infty} k_\alpha(\eta) d\eta d\tilde{r} \qquad (25)$$

EVALUATION OF INTEGRAL I(β) - The integral I(β) has been evaluated for two types of spatial distributions of the species α. In case 1, α was assumed to be confined to the quench layer within which it was uniformly distributed, ie,

$$k_\alpha(\eta_\alpha) = 1: \qquad 0 < \eta_\alpha \leq 1$$

and

$$k_\alpha(\eta_\alpha) = 0: \qquad \eta_\alpha > 1$$

In case 2 concentration of species α was assumed to decay exponentially with distance from the wall, ie:

$$k_\alpha(\eta_\alpha) = e^{-\eta_\alpha} \qquad 0 < \eta_\alpha < \infty$$

The resulting values of I(β) are shown plotted for a range of β in Fig. 3 where it can be seen that the integral is only weakly dependent on the spatial distribution of the species. This useful finding indicates that a detailed knowledge of the diffusion of the unburned hydrocarbons after quenching of the flame front is not essential to a quantitative prediction of their emission rate.

It is now possible to evaluate the mass flow of a species with the aid of Eqs. (17) and

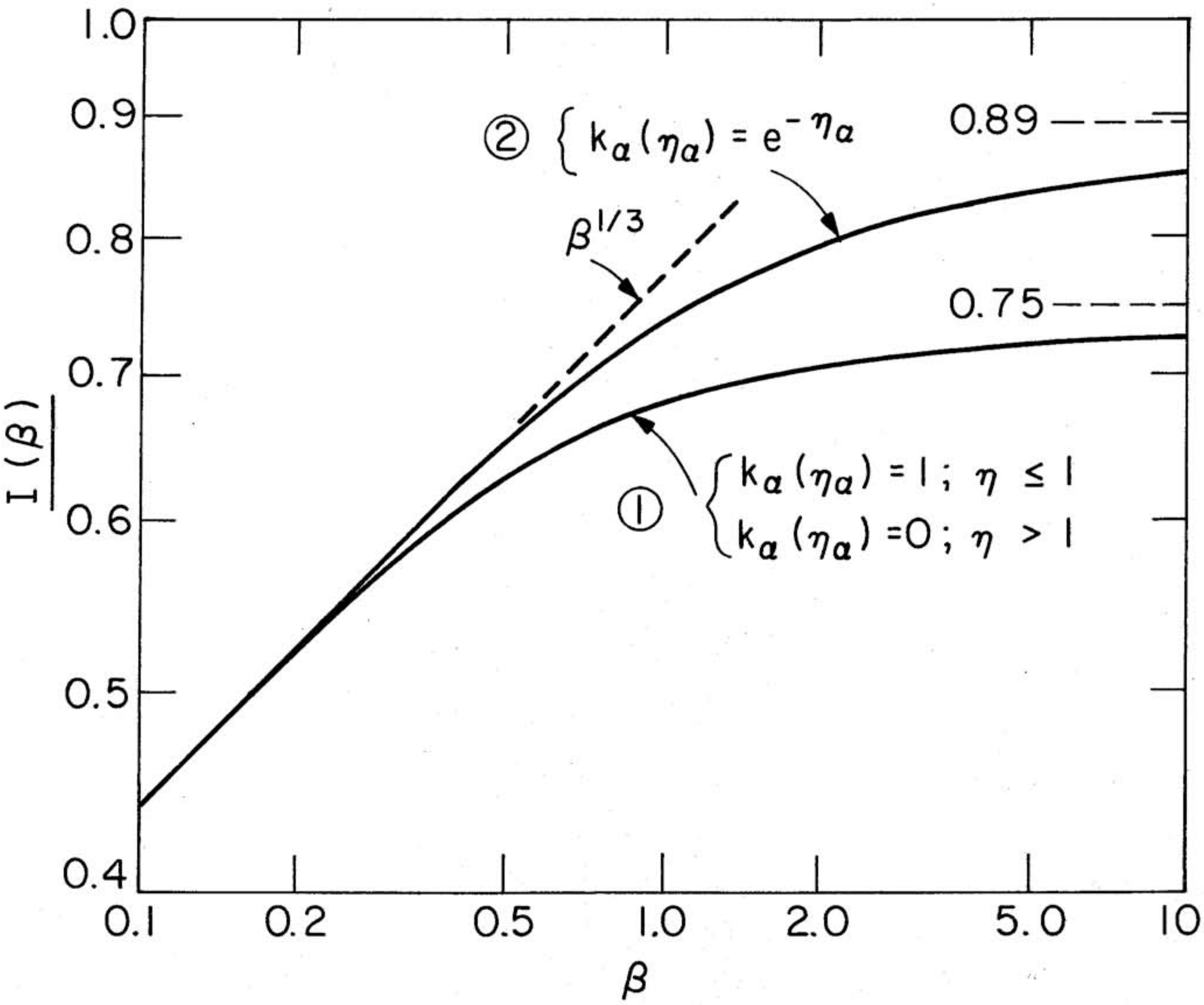

Fig. 3 - Variation of the integral I(β) with β for two distributions of species α.

(24) and the plotted values of $I(\beta)$, using the relation between the total exhaust rate and the Reynolds number given by Eq. (6). Before doing so it is appropriate to briefly discuss the assumptions made in the development of the model.

ASSUMPTIONS - A basic assumption made is that the flow is quasi-steady, ie that the velocity distribution is fully developed at all times. A characteristic diffusion time for the process is $\tau = \delta^2/\nu \approx 40$ms for a boundary layer thickness $\delta = 0.1$ cm and a kinematic viscosity $\nu = 0.25$ cm^2/s. This time is of the same order of magnitude as the exhaust process itself which suggests that the quasi-steady assumption is an approximation, and that the discharge of material from the wall region may be larger than indicated by the model.

The effects of heat transfer between the bulk gas mixture, the quench layer and the confining walls has not been included. Temperature gradients and temporal changes in temperature will affect the flow due to the corresponding changes in viscosity. However, in the final equation (Eq. 24) the viscosity is raised to the 1/6th power indicating that these effects are insignificant. Diffusional mass transfer and mass transfer due to buoyant mixing is not considered to be important as the mass flux from the wall region has been shown to be relatively insensitive to the species distribution (Fig. 3).

Finally, the flow has been assumed incomcompressible. The velocity of sound at 2000°K is approximately 10^5 cm/s while exhaust gas velocities can be expected to be of the order of 10^3 cm/s yielding Mach numbers of $\sim 10^{-2}$.

BLOWDOWN AND EXHAUST OF AN INTERNAL COMBUSTION ENGINE

TOTAL MASS FLOWRATE - The total mass flow rate through the exhaust valve of an internal combustion engine during blowdown and exhaust of the combustion chamber can be calculated using the model introduced by Tabaczynski et al, (3). In this model it is assumed that the combustion products obey the equations of state of a perfect gas with constant specific heats, and that during the initial "blowdown" of the chamber to the exhaust pressure, the flow through the exhaust valve is quasi-steady, adiabatic and isentropic, while during the exhaust stroke following blowdown, the flow is incompressible. Under these conditions the mass flowrate through the valve is given by

$$\dot{m} = C_v A_v (\gamma RT)^{\frac{1}{2}} \left(\frac{P}{RT}\right) \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \qquad (26)$$

when $P/P_e \geq [(\gamma + 1)/2]^{\gamma/(\gamma - 1)}$

$$\dot{m} = C_v A_v (\gamma RT)^{\frac{1}{2}} \left(\frac{P}{RT}\right) \left\{\frac{2}{\gamma - 1} \left[\left(\frac{P}{P_e}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_e}\right)^{\frac{\gamma + 1}{\gamma}}\right]\right\}^{\frac{1}{2}} \qquad (27)$$

when $P/P_e \leq [(\gamma + 1)/2]^{\gamma/(\gamma - 1)}$ and

$$\dot{m} = -\omega \frac{dV}{d\theta} \left(\frac{P_e}{RT}\right) \qquad (28)$$

when $P/P_e = 1$ during the exhaust stroke. In Eqs. (26) - (28):

- C_v = discharge coefficient for valve
- A_v = valve area
- γ = specific heat ratio of exhaust gas
- R = specific gas constant
- T = stagnation temperature in the combustion chamber
- P = stagnation pressure
- P_e = pressure at exhaust valve exit
- ω = angular speed of engine
- θ = ωt = crank angle degrees ATC
- $V(\theta)$ = combustion chamber volume

If we now use the assumption made in our model of quench layer entrainment that the flow within the combustion chamber is incompressible, then the total mass in the chamber is approximately given by

$$M = \rho V \qquad (29)$$

when

$$\rho = P/RT \qquad (30)$$

is the stagnation density.

Differentiating (29) with respect to time we obtain

$$\dot{M} = -\dot{m} = \dot{\rho} V + \rho \dot{V} \qquad (31)$$

In most practical cases the maximum mass flowrate during blowdown occurs near BDC and its duration t_b is small compared to that of the exhaust stroke. Under these conditions the term $\dot{V}$ in Eq. (31) may be neglected and we have for $t_4 < t < t_4 + t_b$

$$\dot{m} \approx -\dot{\rho} V_{BDC} \qquad (32)$$

where t_4 is the time the exhaust valve opens. On the other hand during the exhaust stroke $\rho = \rho_e = P_e/RT_e \approx$ constant and for $t_4 + t_b < t$ Eq. (31) becomes identical to Eq. (28).

Using the relations

$$\frac{P}{P_e} = \left(\frac{\rho}{\rho_e}\right)^{\gamma} = \left(\frac{T}{T_e}\right)^{\gamma/(\gamma - 1)} \qquad (33)$$

for the isentropic expansion of a perfect gas and introducing the dimensionless variables

$$\tilde{m} = m/M_4$$
$$\tilde{V} = V/V_{BDC}$$
$$\tilde{\rho} = \rho/\rho_e$$
$$\tilde{\rho}_c = [(\gamma + 1)/2]^{1/(\gamma - 1)},$$

Eqs. (26) - (28) and (32) can be written in the dimensionless form

$$\tilde{\rho}_4 \frac{d\tilde{m}}{d\theta} = -\frac{d\tilde{\rho}}{d\theta} = \tilde{\rho}_c \tilde{A}_v \left(\frac{\tilde{\rho}}{\tilde{\rho}_c}\right)^{(\gamma + 1)/2} \qquad (34)$$

when

$$\theta_4 \leq \theta \leq \theta_4 + \theta_b \text{ and } \tilde{\rho} \leq \tilde{\rho}_c,$$

$$\tilde{\rho}_4 \frac{d\tilde{m}}{d\theta} = -\frac{d\tilde{\rho}}{d\theta} = \tilde{\rho}_c \tilde{A}_v \left(\frac{\tilde{\rho}^{\gamma - 1} - 1}{\tilde{\rho}_c^{\gamma - 1} - 1}\right) \qquad (35)$$

when $\theta_4 \leq \theta \leq \theta_4 + \theta_b$ and $\tilde{\rho}_c \geq \tilde{\rho} \geq 1$,

$$\tilde{\rho}_4 \frac{d\tilde{m}}{d\theta} = -\frac{d\tilde{V}}{d\theta} \qquad (36)$$

when $\theta_4 + \theta_b \leq \theta$, when

$$\tilde{A}_v = C_v A_v a_4 / V_4 \; \omega \tilde{\rho}_c \tilde{\rho}_4^{(\gamma - 1)/2} \qquad (37)$$

in which

$$a_4 = (\gamma R T_4)^{\frac{1}{2}}$$

is the sound speed at t_4.

Given the valve area $A_v(t)$ and the cylinder volume $V(t)$ as functions of the time, Eqs. (34) - (36) can be integrated numerically to obtain $\tilde{\rho}$ and $d\tilde{m}/d\theta$ as functions of θ.

An excellent analytic approximation may easily be obtained, however, by observing that in most cases of practical interest $[(\gamma - 1)/2] \times |\ln \tilde{\rho}/\tilde{\rho}_c| << 1$ so that

$$\left(\frac{\tilde{\rho}}{\tilde{\rho}_c}\right)^{\frac{\gamma + 1}{2}} \approx \frac{\tilde{\rho}}{\tilde{\rho}_c}$$

and

$$\left(\frac{\tilde{\rho}^{\gamma - 1} - 1}{\tilde{\rho}_c^{\gamma - 1} - 1}\right)^{\frac{1}{2}} \approx \left(\frac{\tilde{\rho} - 1}{\tilde{\rho}_c - 1}\right)^{\frac{1}{2}}$$

If we further assume that

$$A_v \approx (dA_v/d\theta)_4 \; (\theta - \theta_4) \qquad (38)$$

when $\theta_4 \leq \theta \leq \theta_4 + \theta_b$ and

$$V = V_{BDC}[r_c^{-1} + \tfrac{1}{2}(1 - r_c^{-1})(1 - \cos\theta)], \qquad (39)$$

when $\theta_4 + \theta_b \leq \theta$ and introduce the new variable

$$\tilde{\theta} = (\theta - \theta_4)/\theta_v \qquad (40)$$

where

$$\theta_v = \left[\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \left(\frac{C_v a_4}{V_4 \omega}\right) \left(\frac{dA_v}{d\theta}\right)_4\right]^{-\frac{1}{2}}, \qquad (41)$$

then Eqs. (34) - (36) can be integrated to give

$$\tilde{\rho} = \tilde{\rho}_4 \; e^{-\tilde{\theta}^2/2} \qquad (42a)$$

$$d\tilde{m}/d\tilde{\theta} = \tilde{\theta} \; \tilde{\rho}/\tilde{\rho}_4 \qquad (42b)$$

when $\tilde{\theta}^2 \leq 2\ln \tilde{\rho}_4/\tilde{\rho}_c$,

$$\tilde{\rho} = 1 + \tilde{\rho}_c^2(\tilde{\theta}_b^2 - \tilde{\theta}^2)^2/16(\tilde{\rho}_c - 1) \qquad (43a)$$

$$d\tilde{m}/d\tilde{\theta} = \tilde{\theta}[(\rho - 1)/(\rho_c - 1)]^{\frac{1}{2}} \; \tilde{\rho}_c/\tilde{\rho}_4 \qquad (43b)$$

when $2\ln \tilde{\rho}_4/\tilde{\rho}_c \leq \tilde{\theta}^2 \leq \tilde{\theta}_b^2$ and

$$\tilde{\rho} = 1 \qquad (44a)$$

$$d\tilde{m}/d\tilde{\theta} = -(\sin\theta)(1 - r_c^{-1})\theta_v \pi/360 \; \tilde{\rho}_4 \qquad (44b)$$

when $\tilde{\theta}_b^2 \leq \tilde{\theta}^2$, where

$$\tilde{\theta}_b^2 = 4(1 - \rho_c^{-1}) + 2\ln \tilde{\rho}_4/\tilde{\rho}_c \qquad (45a)$$

when $\tilde{\rho}_4 \leq \tilde{\rho}_c$ and

$$\tilde{\theta}_b^2 = 4(1 - \rho_c^{-1})[(\tilde{\rho}_4 - 1)/(\tilde{\rho}_c - 1)]^{\frac{1}{2}} \qquad (45b)$$

when $\tilde{\rho}_c \geq \tilde{\rho}_4$.

Equations (42) - (44) give the stagnation density in the combustion chamber and total mass flowrate through the exhaust valve as explicit functions of the crank angle. The stagnation pressure and temperature may be obtained from the relations (33) which in terms of our dimensionless variables are simply

$$\tilde{P} = \tilde{\rho}^{\gamma} = \tilde{T}^{\gamma/(\gamma - 1)} \qquad (46)$$

HYDROCARBON MASS FLOWRATE - Using Eqs. (42) - (44) for the total mass flowrate and stagnation density, the hydrocarbon emissions and swept length may now be calculated using the model of quench layer entrainment developed in Section II. Assuming the flow in the cylinder to be symmetrical about the vertical axis in Fig. 1, the Reynold number obtained from Eq. (6) is

$$Re = \frac{\omega}{\pi \ell \rho \nu} \frac{dm}{d\theta} \qquad (47)$$

Substituting this expression into the integrals in Eqs. (18b), (23) and (24) and assuming $\rho\nu \approx \rho_4 \nu_4$ we obtain the swept length

$$r_s(\theta) = \left(\frac{4V_4}{\pi \ell}\right)^{\frac{1}{2}} \left[\frac{f''(o)}{\sqrt{2}} \frac{\delta_{q4}}{\delta_{v4}} F\left(3/2, \frac{\theta - \theta_4}{\theta_v}\right)\right]^{1/3} I(\beta) \qquad (48)$$

where

$$\delta_{v4} = (\nu_4 \theta_v/\omega)^{\frac{1}{2}} \tag{49}$$

is a characteristic viscous boundary layer thickness, δ_{q4} is the characteristic thickness of the HC rich quench layer at the time the exhaust valve opens, $I(\beta)$ is the function defined by Eq. (25) and plotted in Fig. 3,

$$\beta(\theta) = [F(1,\frac{\theta - \theta_4}{\theta_v})]^{3/2} [\frac{f''(o)}{\sqrt{2}} \frac{\delta_{q4}}{\delta_{v4}} F(3/2,\frac{\theta - \theta_4}{\theta_v})]^{-1} \tag{50}$$

and

$$F(n,\chi) = \int_0^{\chi} \frac{\tilde{\rho}_4}{\tilde{\rho}} (\frac{1}{2} \frac{dm}{d\theta})^n \, d\theta \tag{51}$$

Finally substituting Eq. (48) into Eq. (17) we obtain the mass of hydrocarbons swept from the wall on one side of the exhaust valve

$$m_{HC}(\theta) = \rho_{HC}\delta_q (\frac{4}{\pi} V_4 \ell)^{\frac{1}{2}} \times [\frac{f''(o)}{\sqrt{2}} \frac{\delta_{q4}}{\delta_{v4}} F(3/2\frac{\theta - \theta_4}{\theta_v})]^{1/3} I(\beta). \tag{52}$$

Note that product $\rho_{HC}\delta_q$ is just the mass of hydrocarbons per unit area of the swept wall and is a constant during the expansion stroke whereas the thickness of the quench layer δ_q increases during expansion as the density decreases.

In principle an explicit equation for the HC mass flowrate $dm_{HC}/d\theta$ may be obtained by differentiating Eq. (52) with respect to θ, however, the resulting expression is quite complicated and in practice it is much simpler to obtain $dm_{HC}/d\theta$ numerically or graphically. The HC mass fraction in the exhaust is then given by the ratio of the HC mass flowrate to the total mass flowrate, ie,

$$f_{HC}(\theta) = (\frac{dm_{HC}}{d\theta})/(\frac{dm}{d\theta}). \tag{53}$$

In the application of the above equations, it should be kept in mind that it has been assumed in our model that the flow through the exhaust valve may be approximated as two dimensional and that the walls on either side of the valve opening extend to infinity. Thus if the swept length becomes comparable with the radius of the exhaust valve or the distance to the side walls of the cylinder, geometrical corrections are necessary.

It should be further kept in mind that the model does not include consideration of the reduction in hydrocarbons due to oxidation either in the cylinder during the expansion stroke or in the flow through the exhaust valve. In this respect the model may be regarded as giving an upper bound to the HC emitted.

CALCULATIONS FOR CFR ENGINE - Illustrative calculations have been carried out for a typical single cylinder CFR engine of the type used by Tabaczynski et al (3) in their experimental studies of mass flowrate and HC emissions. The pertinent characteristics of this engine are summarized in Table 1. Although the engine was run unthrottled in the experiments giving a pressure ratio $\tilde{P}_4 = P_4/P_e = 4.5$ across the exhaust valve at the opening angle, the calculations were made for several values of $\tilde{P}_4$ to illustrate the effect of throttling.

Table 1 - Summary of CFR Engine Parameter and Assumed Operating Conditions

Bore	D	= 3.25 in
Stroke	S	= 4.50 in.
Displacement Volume	V_D	= 37.33 in^3
Compression Ratio	r_c	= 7
Exhaust Valve Diameter	d_v	= 1.25 in
Exhaust Valve Lift	L_v	= 0.30 in
Exhaust Valve Open	θ_4	= 130° ATC
Exhaust Valve Close	θ_{v_c}	= 360° ATC
Discharge Coefficient	C_v	= 0.8
Fuel: Unlead Gasoline		
Correct F/A Mass Ratio	F_c	= .067
F/A Equivalence Ratio	ϕ	= 1.2
Engine Speed	N	= 1200 RPM
Inlet Temperature	T_1	= 350°K
Wall Temperature	T_w	≈ 380°K
Gas Temperature at θ_4	T_4	≈ 1600°K
Inlet Pressure	P_1	≈ 14 psia
Exhaust Pressure	P_e	≈ 14 psia
Pressure Ratio at θ_4	P_4/P_e	≈ 4.5
Effective Specific Heat Ratio	γ	≈ 1.3
Sound Speed at T_4	α_4	≈ 2000 ft/sec
Characteristic Angle (Eq. 41)	θ_v	≈ 40°

The stagnation density ρ and total mass flowrate $dm/d\theta$ were calculated from Eqs. (42) - (44). To calculate the HC emissions values of the quench layer thickness δ_q and kinematic viscosity ν are required.

The quench layer thickness was estimated from the equation

$$\delta_q = 0.4 \, \delta_{qr} (P_r/P)^{\alpha} (T_r/T_u)^{\beta} \tag{54}$$

where T_u is the unburned gas temperature and the parameters α, β, δ_{qr}, P_r and T_r obtained from data in references (6) - (9) are summarized in Table 2. Assuming isentropic compression of the unburned gas from the inlet temperature T_1 and pressure P_1 through a pressure ratio $P/P_1 \approx 25$ at the time the quench layer is formed and using the perfect gas relation $P_4/P_1 = T_4/T_1$, we obtain from equation (54) and the data in Tables (1) and (2) the value

$$\delta_q \approx .011\ \tilde{P}_4^{-.66}\text{cm.} \tag{55}$$

The corresponding value at the time the exhaust valve opens was assumed to be

$$\delta_{q4} \approx r_c\delta_q \approx .077\ \tilde{P}_4^{-.66}\text{cm.} \tag{56}$$

Table 2 - Quenching Parameters as a Function of Equivalence Ratio ϕ

$T_r = 373°K$, $P_r = 4$ atm.

ϕ	δ_{q_r}, cm	α	β
0.70	.085	0.53	0.64
1.00	.064	0.52	0.50
1.10	.056	0.62	0.50
1.20	.054	0.66	0.50
1.30	.054	0.66	0.50

The kinematic viscosity was calculated from the equation

$$\nu_4 = \nu_o\left(\frac{P_o}{P_4}\right)\left(\frac{T_4 + T_w}{2T_o}\right)^{3/2} \tag{57}$$

where T_w is the wall temperature, $\nu_o = 0.15$ cm^2/sec, $P_o = 1$ atm and $T_o = 288°K$. Substituting Eq. (57) into Eq. (49) and using the data in Table 1, we obtain the boundary layer thickness

$$\delta_{v4} \approx .086\ \tilde{P}_4^{-0.5}\text{cm} \tag{58}$$

Finally substituting Eqs. (56) and (58) into Eqs. (48), (50) and (52) and setting $\ell = \pi d_v$ and $V_4 \approx \frac{\pi}{4} D^2S$ we obtain

$$\frac{m_{HC}}{\rho_{HC}\delta_q A} = \frac{r_s}{R}\left(\frac{4d_v}{D}\right) = \frac{8}{\pi}\left(\frac{S}{D}\right)^{\frac{1}{2}}\left(\frac{\pi d_v}{D}\right)^{\frac{1}{2}} \times \left[F\left(3/2, \frac{\theta - \theta_4}{\theta_v}\right)\right]^{1/3} I(\beta) \tag{59}$$

in which $A = \pi R^2$ is the area of the head wall, $R = D/2$ is the cylinder radius, d_v is the exhaust valve diameter, S is the stroke,

$$\beta = \left[F\left(1, \frac{\theta - \theta_4}{\theta_v}\right)\right]^{3/2}\left[F\left(3/2, \frac{\theta - \theta_4}{\theta_v}\right)\right]^{-1} \tag{60}$$

and the function $I(\beta)$ is shown in Fig. (3) for two assumed quench layer profiles. In deriving Eqs. (59) we have used the value $f''(o) = 2(2/3)^{\frac{1}{2}}$ obtained differentiating Eq. (4) and neglected the weak pressure dependence of the factor $\tilde{P}_4^{-0.16}$ in the ratio $\delta_{q4}/\delta_{v4} = 0.9\ \tilde{P}_4^{-0.16}$. We have also assumed that geometrical effects associated with the fact that the exhaust valve is circular rather than linear cancel to first order when the HC contribution from the walls on either side of the opening are added and simply doubled the single wall result given by Eq. (52) to obtain the total HC exhausted.

It may be noted that in the limit $\beta \ll 1$, $I(\beta) \propto \beta^{1/3}$ and Eq. (59) becomes

$$\frac{m_{HC}}{\rho_{HC}\delta_q A} = \frac{r_s}{R}\left(\frac{4d_v}{D}\right) = \frac{4}{\pi}\left(\frac{S}{D}\right)^{\frac{1}{2}}\left(\frac{\pi d_v}{D}\right)^{\frac{1}{2}}\left[F\left(1, \frac{\theta - \theta_4}{\theta_v}\right)\right]^{\frac{1}{2}} \tag{59a}$$

The integrals $F(1,\chi)$ and $F(3/2,\chi)$ defined by Eq. (51) can be evaluated using the Eqs. (42)-(44) to obtain $\tilde{p}$ and $d\tilde{m}/d\tilde{\theta}$. Although this can be done, at least in part, analytically the resulting expressions are quite complicated and it is far simpler just to integrate Eq. (51) numerically.

The calculated total mass flowrate divided by the initial mass in the cylinder and the HC mass flowrate divided by the mass of HC in the head wall quench layer are shown as functions of the crank angle in Fig. 4 for several values of the pressure ratio P_4/P_e. Also shown are the experimental measurements of Tabaczynski et al for a pressure ratio $P_4/P_e = 4.5$. Allowing for the relatively low resolution of the experiments ($\gtrsim 14$ CA°) the agreement with the calculated results is remarkably good.

It can be seen that for values of $P_4/P_e >$ 1.1 most of the unburned HC leave the cylinder during blowdown while for $P_4/P_e < 1.1$, most leave during the exhaust stroke. It can also be seen that for $P_4/P_e \lesssim 1.8$, there is an interval following blowdown in which the total mass flow reverses and mass enters the cylinder from the exhaust. During this interval it is assumed that the flow will detach from the cylinder wall as it enters the chamber and the quench layer will be left relatively undistributed. Although any HC already in the exhaust pipe will flow back into the cylinder during this time, it will be carried out when the flow again reverses and the net effect will be zero.

The mass fraction of HC in the exhaust divided by the mass fraction $m_F/(m_F + m_A)$ in the unburned gas is shown as a function of crank angle in Fig. 5. Note that for $P_4/P_e > 1$ this ratio goes to 1 at valve open. Again allowing for low resolution, the agreement between theory and experiment during blowdown is considered good. During the exhaust stroke the measured concentrations are somewhat higher than those predicted but this has a negligible effect on the HC emissions since the total mass flowrate is small at this time.

The integrated mass of HC exhausted divided by the mass of HC in the head wall quench layer is shown as a function of crank angle in Fig. 6.

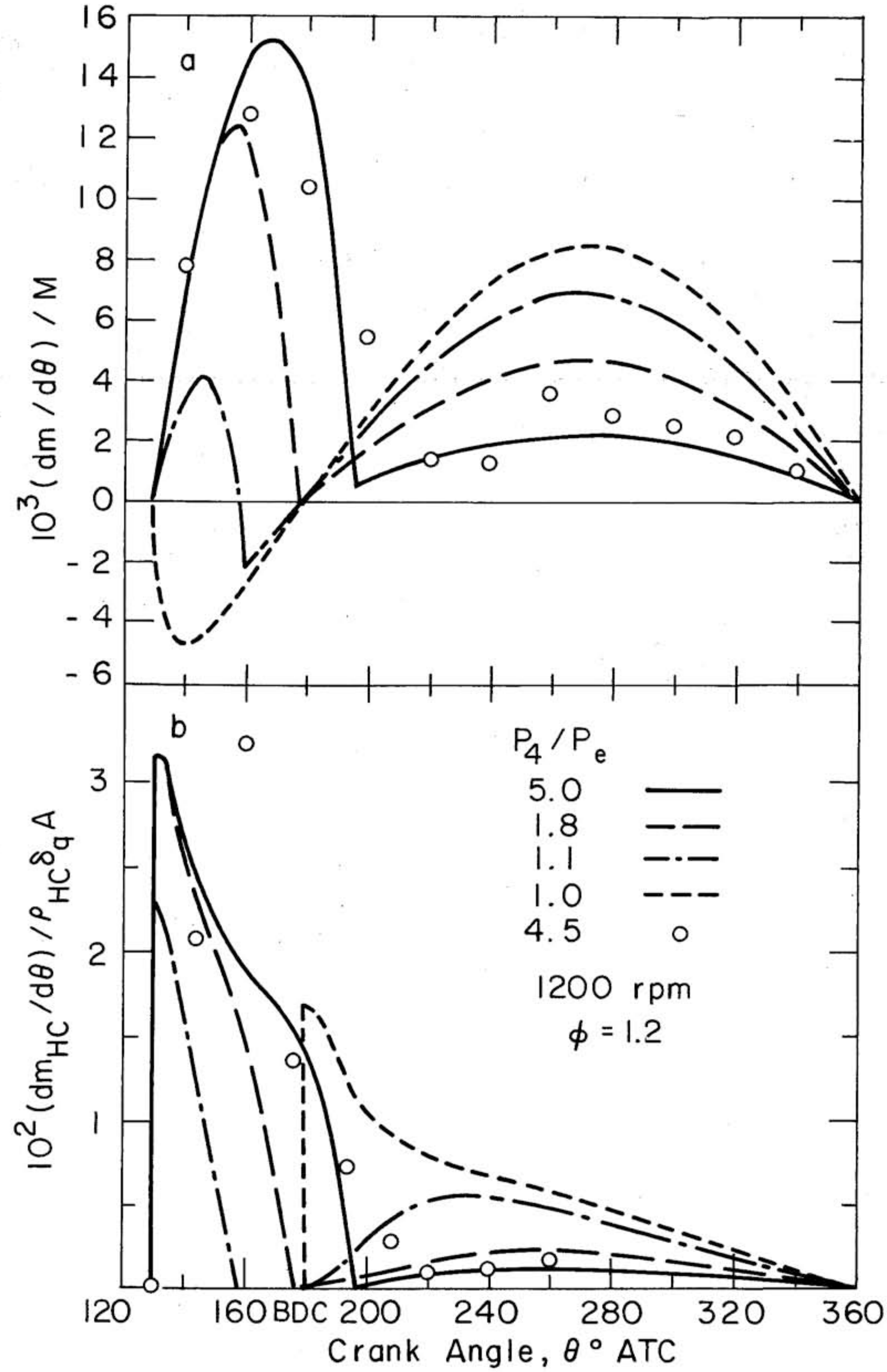

Fig. 4 - (a) Total mass flowrate $dm/d\theta$ divided by original mass M in the cylinder and (b) HC mass flowrate divided by HC mass $\rho_{HC}\delta_q A$ in the head wall quench layer as a function of crank angle for several values of the initial exhaust pressure ratio P_4/P_e. The calculations were made for a typical CFR engine running at 1200 ppm with a fuel/air equivalence ratio $\phi = 1.2$. The points show the experimental data of Tabaczynski, Heywood and Keck (Ref. 3). The experimental resolution in estimated to be of the order of ± 10°CA.

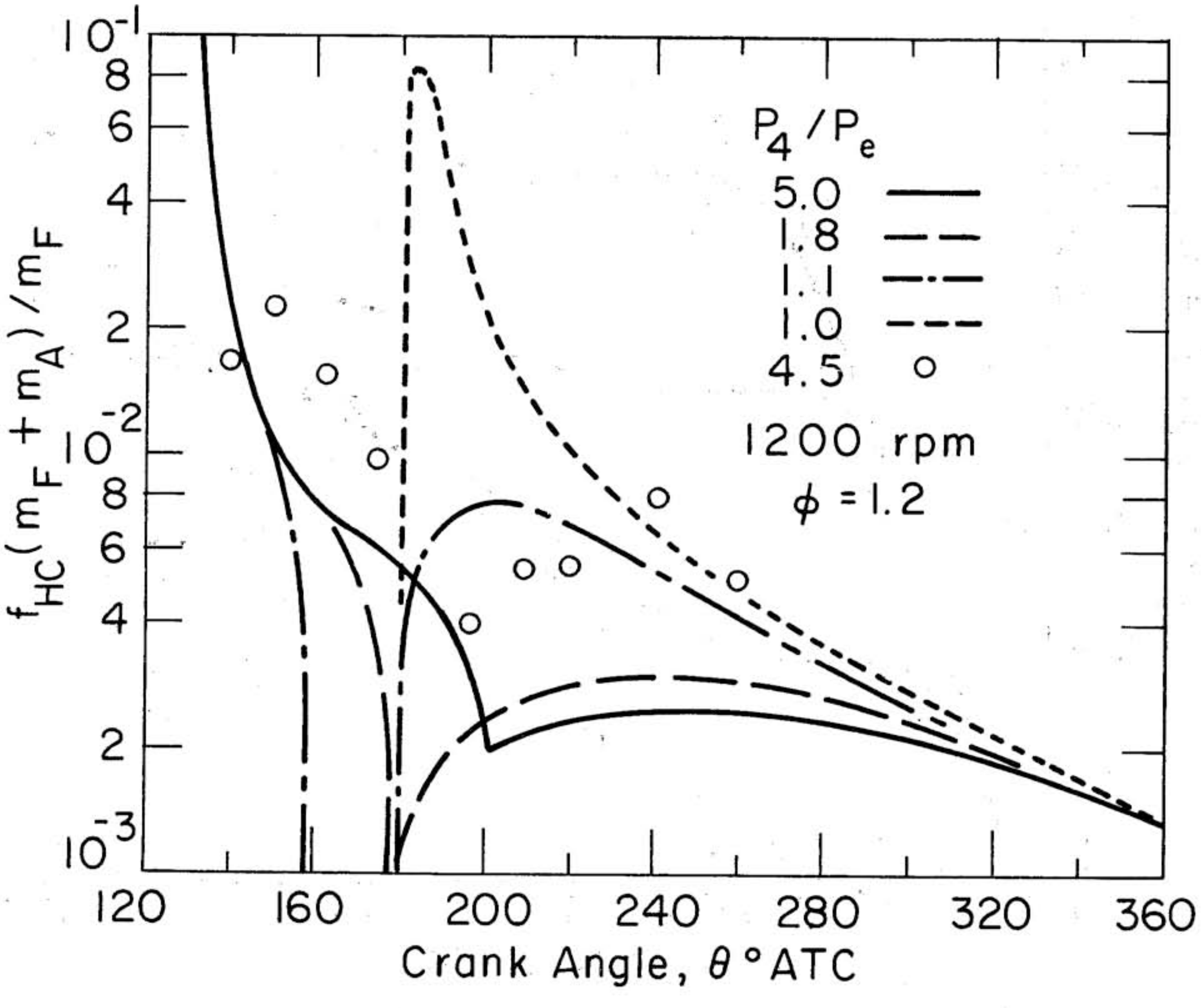

Fig. 5 - HC mass fraction f_{HC} in the exhaust stream divided by HC mass fraction $m_F/(m_F + m_A)$ in the original charge as a function of crank angle for the same conditions as Fig. 4. The experimental points are from Ref. 3.

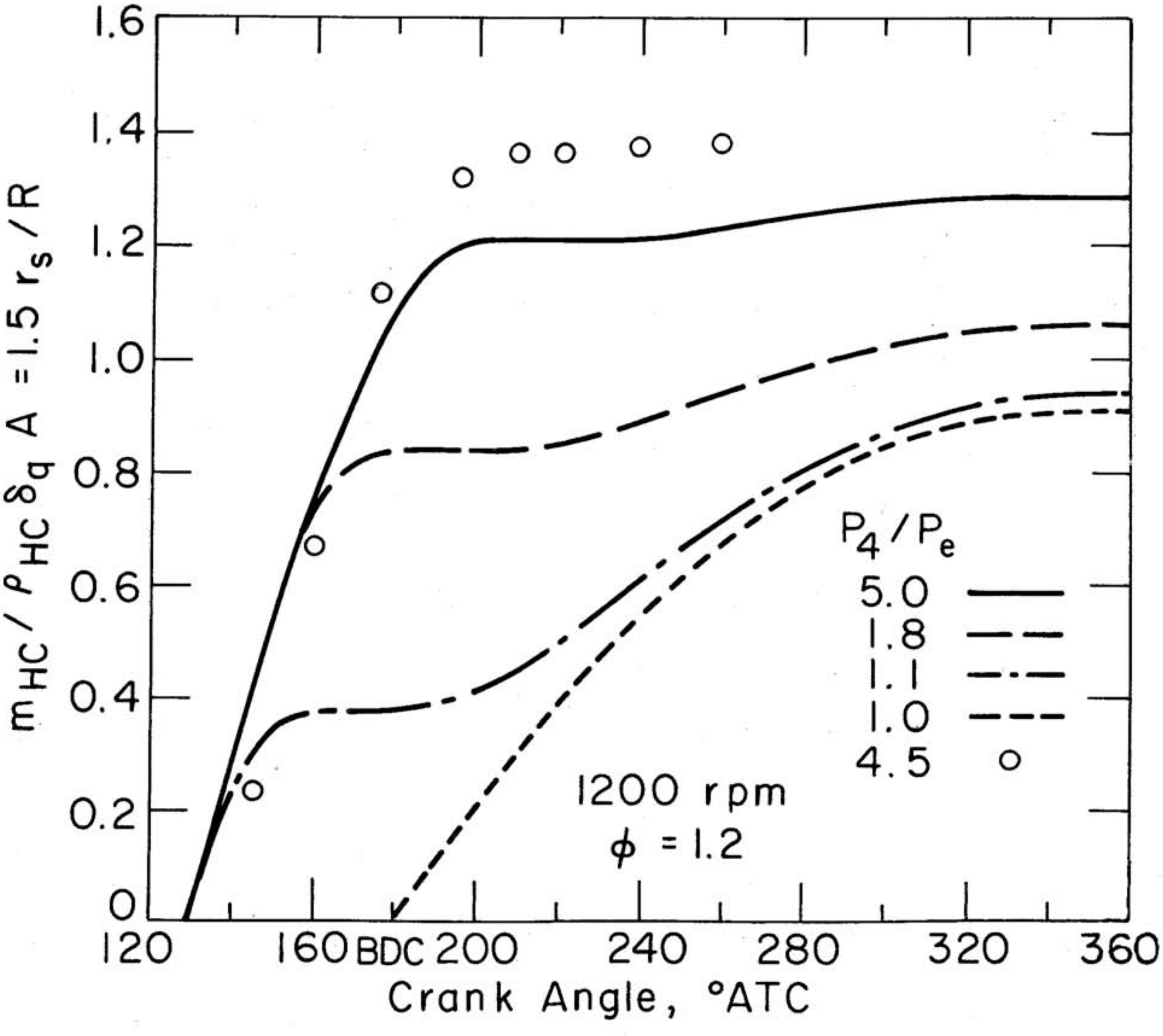

Fig. 6 - HC mass M_{HC} exhausted divided by HC mass $\rho_{HC}\delta_q A$ in the head wall quench layer as a function of crank angle for the same conditions as Fig. 4. Note this figure also gives 1.5 times the ratio of the swept length r_s to the cylinder radius R. The experimental points are from Ref. 3.

Note that this ratio is equal to 1.5 times the ratio of the swept length to the cylinder radius. The model predicts that for all values of P_4/P_e the swept length at the end of the exhaust stroke will be comparable to the cylinder radius and that essentially all the HC in the head wall quench layer will be exhausted. At a pressure ratio $P_4/P_e = 5$, the model predicts HC emission in excess of the HC mass in head wall quench layer implying a possible contribution from the side wall quench layers. The experimental data appear to support this conclusion, although it is somewhat doubtful if either the calculation or the experiments are sufficiently accurate to be certain of this point. As implied by previous comparisons the agreement between the calculated and measured HC emissions is quite good.

The fraction of the quench layer HC exhausted during blowdown is shown as a function of P_4/P_e in Fig. 7. It can be seen that this fraction approaches 1 extremely rapidly for $P_4/P_e > 1$ This is, of course, due to the fact that the rate of quench layer entrainment is very large in the initial flow through the valve.

Scaling of these results in Figs. (4) - (7) to other engines and other operating conditions can easily be done by refering back to the basic equations (48) and (52). In this connection it may be observed, that the dimensionless ratios $m_{HC}/\rho_{HC}\delta_q A$ and r_s/R vary only as the cube root of the ratio δ_{q4}/δ_{v4} and the square root of the ratio S/D and d_v/D and are therefore relatively insensitive to changes either in quench layer thickness, engine geometry or operating conditions.

APPLICATION TO WANKEL ENGINES - The results given above for a CFR engine may also be used to predict the quench layer HC swept from the rotor face in a Wankel engine. The only change necessary, apart from the trivial recalculation of the scaling factors $(4V_4/\pi\ell)^{1/2}$ and $(4V_4\ell/\pi)^{1/2}$ in Eqs. (48) and (52), is to multiply the integral in Eq. (51) by a factor of 2. This because the

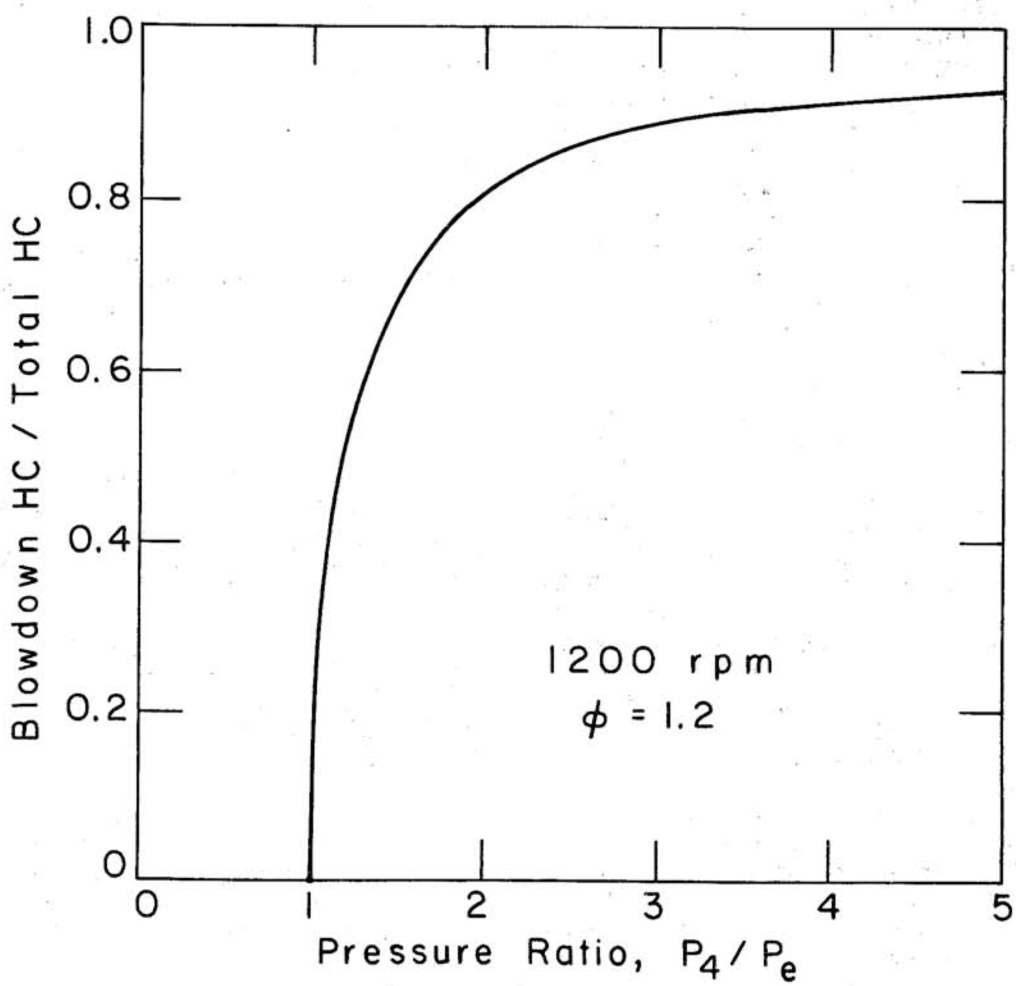

Fig. 7 - Fraction of HC mass exhausted during blowdown as a function of the initial exhaust pressure ratio P_4/P_e for the same conditions as Fig. 4.

exhaust valve in a Wankel is effectively located in a corner and the Reynolds No. obtained from Eq. (6) in this case is

$$R_e = \frac{2\omega}{\pi \ell \rho \nu} \frac{dm}{d\theta} , \qquad (60)$$

which is twice the the value given by Eq. (47).

For an unthrottled Mazda engine of the type used in the experiments of Ferguson, Danieli, Heywood and Keck (10) on HC emissions from Wankel engines, it is found that ratio of the swept length to the length of the rotor at the end of blowdown is close to 1/2. Values of this ratio for other conditions may be scaled from Fig. 6.

Since there is no exhaust valve in a Wankle engine, it is not possible (with the resolution of present systems) to separate the HC exhausted during blowdown from that exhausted at the end of the previous stroke. Thus a direct comparison with experiment cannot be made in this case. Instead the model may be used to predict the quench layer HC exhausted.

SUMMARY AND CONCLUSIONS - A model of quench layer entrainment during blowdown and exhaust of the combustion chamber in an internal combustion engine has been developed. The model is based on the assumptions that the flow in the chamber may be treated as 2 dimensional, quasi-steady incompressible and isentropic. It can be used to predict the contribution of the head wall quench layer to the HC emissions from a knowledge of the quench layer thickness, engine geometry and operating conditions.

The essential results of the model are contained in Eq. (48) which shows that the length r_s from which the quench layer is swept is proportional to the square root of the ratio of the combustion chamber volume V_4 at valve open to the length of the exhaust port ℓ and the cube root of the ratio of the quench layer thickness δ_q to the thickness of the viscous boundary layer thickness δ_v based on the exhaust time. As a consequence r_s is relatively insensitive to changes in engine size or operating conditions. For a typical CFR engine, the value of r_s at the end of the exhaust stroke is approximately equal to the cylinder radius; for a typical Wankle engine r_s is approximately 1/2 the rotor length.

The total mass of HC exhausted is given by $m_{HC} \approx \rho_{HC} \delta_q \ell r_s$, where ρ_{HC} is the HC density at the time the quench layer is formed near TDC. Dividing this expression by the mass of fuel originally in the combustion chamber, $m_F \approx \rho_{HC} \times V_{TDC}$, and using Eq. (48) the contribution of the quench layer to the mass fraction of the original fuel which leaves the combustion chamber unburned is found to be proportional to $r_c \delta_q^{4/3} (\ell/V_4)^{1/2} / \delta_v^{1/3}$, where r_c is the compression ratio. It can be seen that the most important parameters determining the HC emissions in this case are the quench layer thickness and the compression ratio.

The model can also be used to calculate the time resolved HC mass flowrate through the exhaust. This information is important both for estimating the fraction of the unburned HC which is oxidized during the exhaust process and for interpreting the results of diagnostic experiments on HC emissions such as those in Refs. (3) and (10). It is found that for exhaust pressure ratios at valve open $P_4/P_e > 1.1$ most of the HC are exhausted during the first half of blowdown; for $P_4/P_e < 1.1$ most of the HC are exhausted during the first half of the exhaust stroke.

On the basis of the above results it appears that the most effective method for reducing unburned HC emissions from the head wall quench layer is to reduce the quench layer thickness. Possible methods of acheiving this include charge stratification, wall heating and selection of fuels having intrinsically thin quench layers. It is also possible that the quench layer could be "held up" by a dam around the exhaust valve. Preliminary estimates indicate that such a dam would have to be at least an 1/8 inch high to be effective. A more detailed theoretical investigation of this problem is currently being initiated; parallel experimental studies are also being considered.

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