values of the Peclet number is considered.

The equations having been linearized, the solution of the equations must be added:

$$\Delta_2 T^{(v)} = \mathrm{Pe}(v/u) T_y^{(v)}$$

with homogeneous boundary conditions on the y and x axis.

The zero-approximation T_0 is unchanged. For the first approximation, the solution of this equation must be added:

$$\Delta_2 T_1^{(v)} = \frac{v}{u} T_{0y} = \frac{v}{u} \frac{2(1 - T_w)}{\pi} \frac{x}{x^2 + y^2}$$

The method of solving this equation is that already applied. The result is the following:

$$T_1^{(v)} = -\frac{(1 - T_w)}{\pi} \frac{v}{u} y \tan^{-1} \frac{x}{y}$$

This expression has a sign other than T_1, and hence the influence of a fluid injection is a virtual decrease of Peclet number. This is seen better by examining $T_y^{(v)}$.

At the wall

$$(T_{1,y}{}^v)_{y=0} = -\frac{1 - T_w}{2\pi} \frac{v}{u}$$

and remembering the (constant) value of $T_{1,y}$, it can be concluded that the relative decreasing of the Peclet coefficient in the heat flux expression is $1 - \frac{1}{2}v/u$; besides, for high value of the ratio v/u, this coefficient reaches negative values.

Concluding Remarks

In this paper, the energy equation studying the case in which it has only the Peclet number as a parameter is taken into account. Then solutions by means of an iterative technique for large Peclet number have been presented. An expansion in series has been employed for small value of Peclet number. As it has been possible to obtain the first three approximations in closed form, the discussion of results has been very easy. Finally, the influence of a fluid injection has been taken into account, and in this case the results obtained have also been easy to analyze.

Appendix

It is now shown how it is possible to solve the equation $\Delta_2 T = 0$ with the boundary conditions $T(0, y) = 0$ and $T(x, 0) = x^2$.

The following conformal mapping, $\zeta = \log z^2$, which maps our quadrant in a strip, is used. On the two sides of the strip, the boundary conditions are

$$T(\xi, 0) = e^\xi \quad [A1a]$$

$$T(\xi, \pi) = 0 \quad [A1b]$$

A harmonic function satisfying these conditions is the imaginary part of the following analytic function:

$$(1/\pi) e^\zeta (i\pi - \zeta) \quad [A2]$$

$$T = (e^\xi/\pi)\{(\pi - \eta)\cos\eta - \xi \operatorname{sen}\eta\} \quad [A3]$$

Since $\xi = \ln(x^2 + y^2)$ and $\eta = \tan^{-1} 2xy/(x^2 - y^2)$

$$T = \frac{x^2 + y^2}{\pi}\left\{\left[\pi - \tan^{-1}\frac{2xy}{x^2 + y^2}\right]\cos\left(\tan^{-1}\frac{2xy}{x^2 - y^2}\right) - [\ln(x^2 + y^2)]\operatorname{sen}\left(\tan^{-1}\frac{2xy}{x^2 - y^2}\right)\right\}$$

Nomenclature

Pe = $\rho c_p L u/\lambda$ = Peclet number
u, v = velocity components
T = temperature
z = $(y/2)(x/\mathrm{Pe})^{-1/2}$ or x/y
λ = thermal conductivity

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Race Track Flow Visualization of Hypersonic Wakes[1]

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A technique for photographing high velocity self-luminous airflow in the wakes of hypersonic pellets is described. This method involves the use of a drum camera, an array of slits, and a mirror system to provide separate images on the film for each slit. The drum speed is matched through optical magnification to the approximate velocity of the fluid events occurring in the wake flow. The "tracking" feature allows low intensity events to be exposed on the film without blurring. Sample photographs of laminar and turbulent wakes in argon are presented.

THE RACE track (1)[4] method is a technique of flow visualization which enables self-luminous events to be photographed in the wake of hypersonic pellets when the illumination is too low and the speed is too high for high speed movies and snapshots. The patterns of luminosity obtained by this technique are currently proving of great value for the study of transition from laminar to turbulent flow in wakes. These patterns are also giving information about the structure and growth of both laminar and turbulent wakes.

Race Track Method

The basic principle of the race track method is as follows. An optical system is used to form an image of the moving object on a film mounted in a rotating drum. The drum speed is adjusted to match the image speed so that the image remains stationary with respect to the film. In this way the action is effectively stopped, and much longer exposures may be obtained without blurring of the image. In practice, the exposure is determined by the time t_e it takes the object to cross a slit placed in front of it perpendicular to its motion, and the amount of blurring which occurs due to mis-tracking is then $(\mathbf{v}_i - \mathbf{v}_f)t_e$, where $\mathbf{v}_i$ is the vector velocity of the image and $\mathbf{v}_f$ is the vector velocity of the film. This is the basic relation for determining the permissible exposure.

A schematic diagram of a typical race track setup is shown in Fig. 1. The arrangement reported here consists of 1) an f 2.5 drum camera, 2) a row of three vertical slits 5 mm wide mounted on the test chamber window 15 mm apart, and 3)

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an array of mirrors to preserve the optical path from each slit to the film. Two purposes of the multiple slits are 1) to produce several images for studying the spatial correlation of luminous events as they move downstream from the pellet; and 2) by using slits of different widths to increase the dynamic range over which the luminosity can be recorded. (A slightly different arrangement of the slits and optical system enables orthogonal views of the wake to be obtained.)

In the present work, the system was adjusted to track luminous eddies 5 to 20 body diam downstream of the pellet. The velocity of these eddies determined from the "feather" structure of conventional streak photographs (1,2) was found to be approximately one fifth the pellet velocity. Thus, for pellet velocities in the vicinity of 15,000 fps, the effective exposure times were about 5 μsec. Note that the tracking speed may be varied by adjusting either the film speed or the magnification of the optics. An upper limit to the effective tracking speed is set by the maximum permissible drum speed and the degree of demagnification which can be employed without loss of detail in the image.

Interpretation of Photographs

Some typical race track photographs of both laminar and turbulent wakes behind 5.6 mm diam pellets fired in argon are shown in Fig. 2. The three images were produced by three slits 15 mm apart along the axis of the range. The scales refer to the leading image in each photograph. The shock shape (4) is drawn in for the leading image on the right. Because of the deceleration of the axial velocity v_{ie} of the eddy images, there is a continuous change in the local axial distortion factor v_f/v_{ie} which tends to drag out the wake more farther back. The film speed v_f was set here to track the events occurring 10 body diam downstream of the pellet. Additional photographs taken in xenon are shown in the following note (2). In interpreting these photographs, it is important to keep in mind that they are not snapshots. As in the case of focal plane shutters, different parts of the picture are exposed at different times. This leads to a somewhat peculiar distortion of the photograph, in which, although the apparent length of the wake is shortened, the structure of the eddies that are being tracked is correctly reproduced. The effect can be most easily understood with the aid of an x, t diagram with the coordinate system fixed to the film, the pellet, eddy, and slit images having passed through the lens. Fig. 3 shows a representative diagram of this type. On this x, t diagram, the image of the slit on the diagram moves in the negative x direction at the film velocity v_f while the image of the pellet on the diagram moves in the positive direction with a velocity $(v_{ip} - v_f)$, where v_{ip} is the velocity of the pellet image. In depicting the paths of eddies for illustrative purpose, the simplifying assumption has been made that they are being exactly tracked a short distance behind the pellet. In this case the spatial separation of the eddies on a race track photograph is the same as that which would be seen in a snapshot (i.e., $l_{eRT} = l_{es}$ in Fig. 3, where l_{eRT} is the length between two tracked eddies on the race track film and l_{es} is the length between these same two eddies on an instantaneous snapshot). Note, however, that the separation of the pellet (which is not being tracked) from the eddies is not that which would be seen in a snapshot. It is seen from Fig. 3 that on this x, t diagram the ratio of the apparent separation l_{RT} between a point in the wake and the pellet on a race track photograph to the corresponding separation l_s in a snapshot is $l_{RT}/l_s = v_f/v_{ip}$. The same relation in differential form also describes the local distortion of any element of length parallel to the wake axis if the local velocity is substituted for the pellet velocity. There is no distortion of length in the radial direction. Note that eddies 2 and 3, which are being exactly tracked, are exposed for the time t_e in which they traverse the slit and have the same separation that they would in a snapshot. Also note that eddies 1, 4,

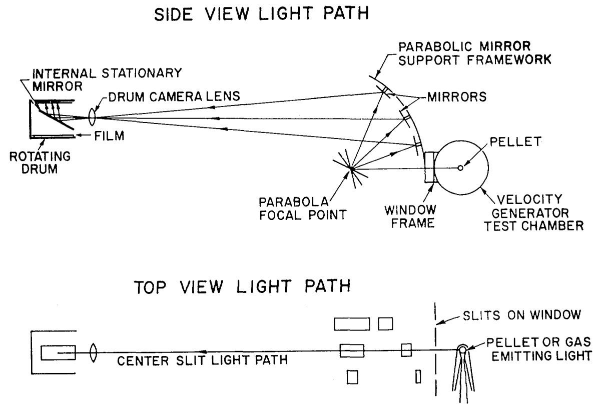

Fig. 1 Typical race track optical layout for obtaining three wake photographs on a single shot

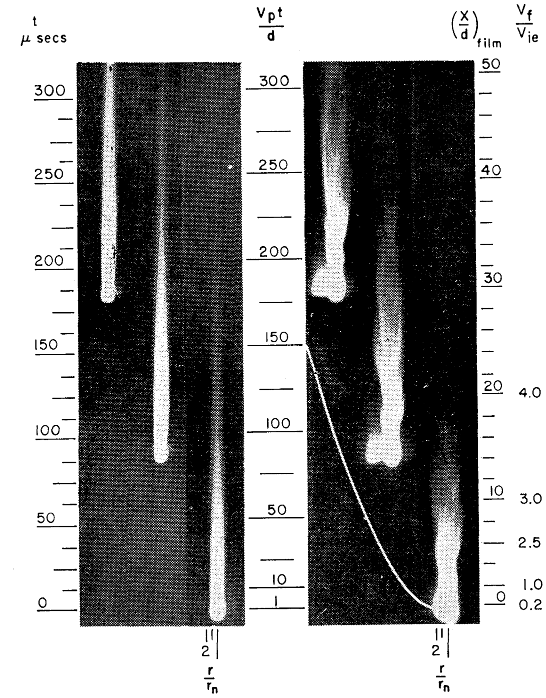

Fig. 2 Race track photographs of laminar (left) and turbulent (right) wakes of hypersonic pellets in argon

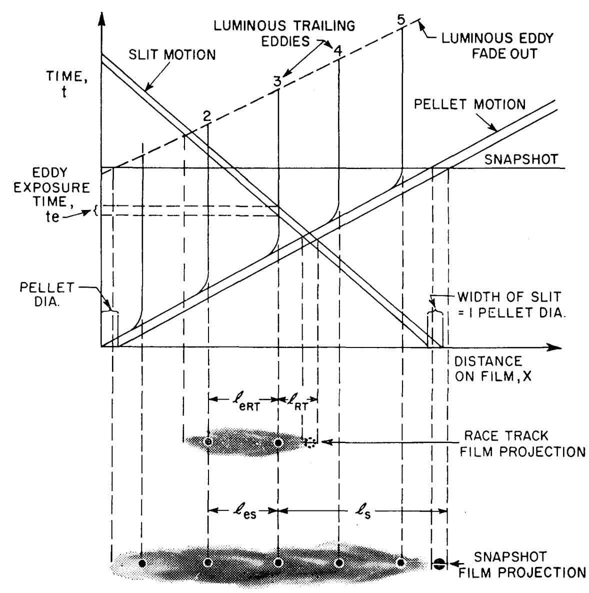

Fig. 3 x, t diagram in film coordinates for a single slit

and 5, which appear in the snapshot, are not seen on the race track photograph.

The scales given in Fig. 2 were constructed with the aid of an x, t diagram like that in Fig. 3 and are designed to assist in the interpretation of the photographs. Shown are 1) the time t after passage of the pellet during which the exposure was made, 2) the distance in body diameters $v_p t/d$ to the pellet at that time, 3) the film axial length scale in body diameters $(x/d)_{\text{film}}$, 4) the radial length scale in body radii r/r_u, and 5) the ratio of tracking speed to particle speed in the wake, v_f/v_{ie} (3). Also shown is the shape that the bow shock (4) would have had if it had been visible. Note that, according to the arguments presented in the preceding paragraph, the factor v_f/v_{ie} on that x, t plane gives the local distortion of distance parallel to the wake axis. Thus, due to the velocity decay of the eddies in the vicinity of the pellet, events in the race track photographs have only 0.2 of their axial separation, whereas at $(x/d)_{\text{film}} = 20$ they have 4 times their separation. A tentative theoretical interpretation of some of the aerodynamic phenomena observed is given in the following note (2).

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Vortex Loops in the Trails Behind Hypervelocity Pellets[1]

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THE PERIODIC shedding of vortices behind circular cylinders and bluff bodies in subsonic flow at low Reynolds number is a well-known phenomenon. (For a summary, see Goldstein, Ref. 1, and Torobin and Gauvin, Ref. 2). Roshko (3)[4] examined in detail the decay of the discrete vortex system behind a cylinder into a fully developed turbulent wake having a continuous energy spectrum. He clearly showed that the generator of and the source of the energy content of the turbulent wake was the vortex system shed by the body. The purpose of this note is to compare photographs of the hypervelocity wake with those of the incompressible wake, and, as a result, to postulate that the shedding of vortices is the source of turbulence in the hypersonic as well as the subsonic wake. When hypervelocity vortex shedding occurs, it must be taken into account in base flow and wake growth studies.

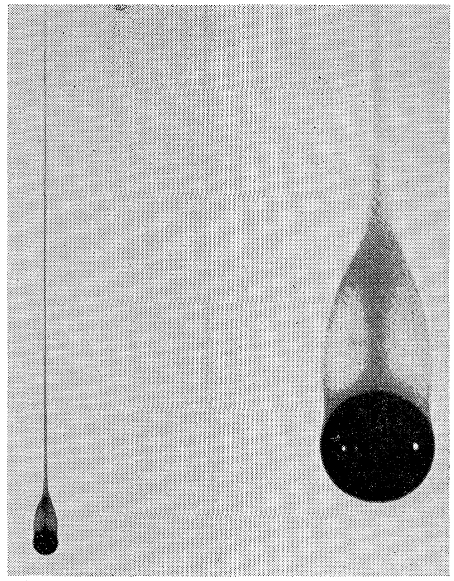

Fig. 1 Single thread incompressible trail. Fluid, water; pellet, CCl_4 with dye; Reynolds no.$_\infty$ (diam) = 0 to 210 (Ref. 6)

The careful experiments of Magarvey and Bishop (4–6) detail incompressible wakes behind immiscible spherical dye drops falling through water. The dye traces the fluid that streams off the body, allowing examination of the motion set up in the wake. Instantaneous photographs of these incompressible wakes are presented in each reference cited. Six distinct regimes of flow for the incompressible trail can be delineated (6): class I, single thread; II, double thread; III, double thread with waves; IV, procession of vortex

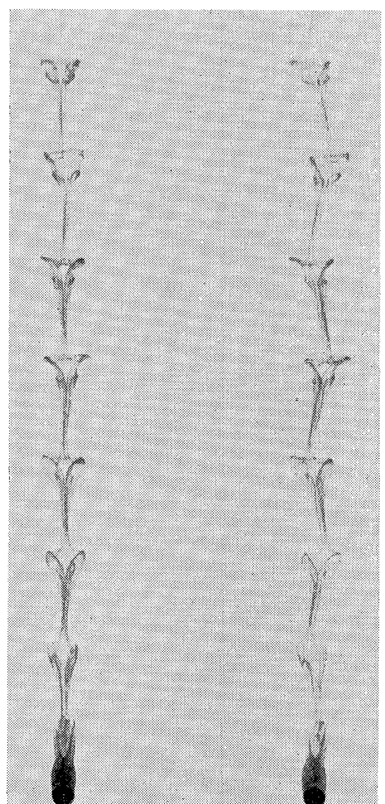

Fig. 2 Vortex loop incompressible trail (stereo pair). Fluid, water; pellet, CCl_4 with dye; Reynolds no.$_\infty$ (diam) = 290 to 410 (Ref. 5)

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